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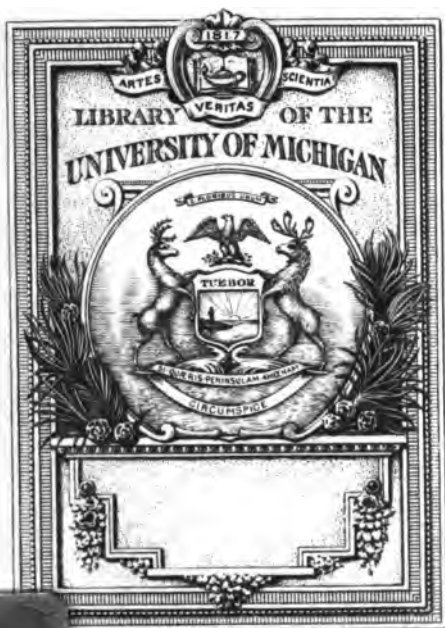
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The Gift of
WILLIAM H. BUTTS, Ph.D.
A.B. 1878 A.M. 1879
Teacher of Mathematics
1898 to 1922
Assistant Dean, College of Engineering
1908 to 1922
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1922



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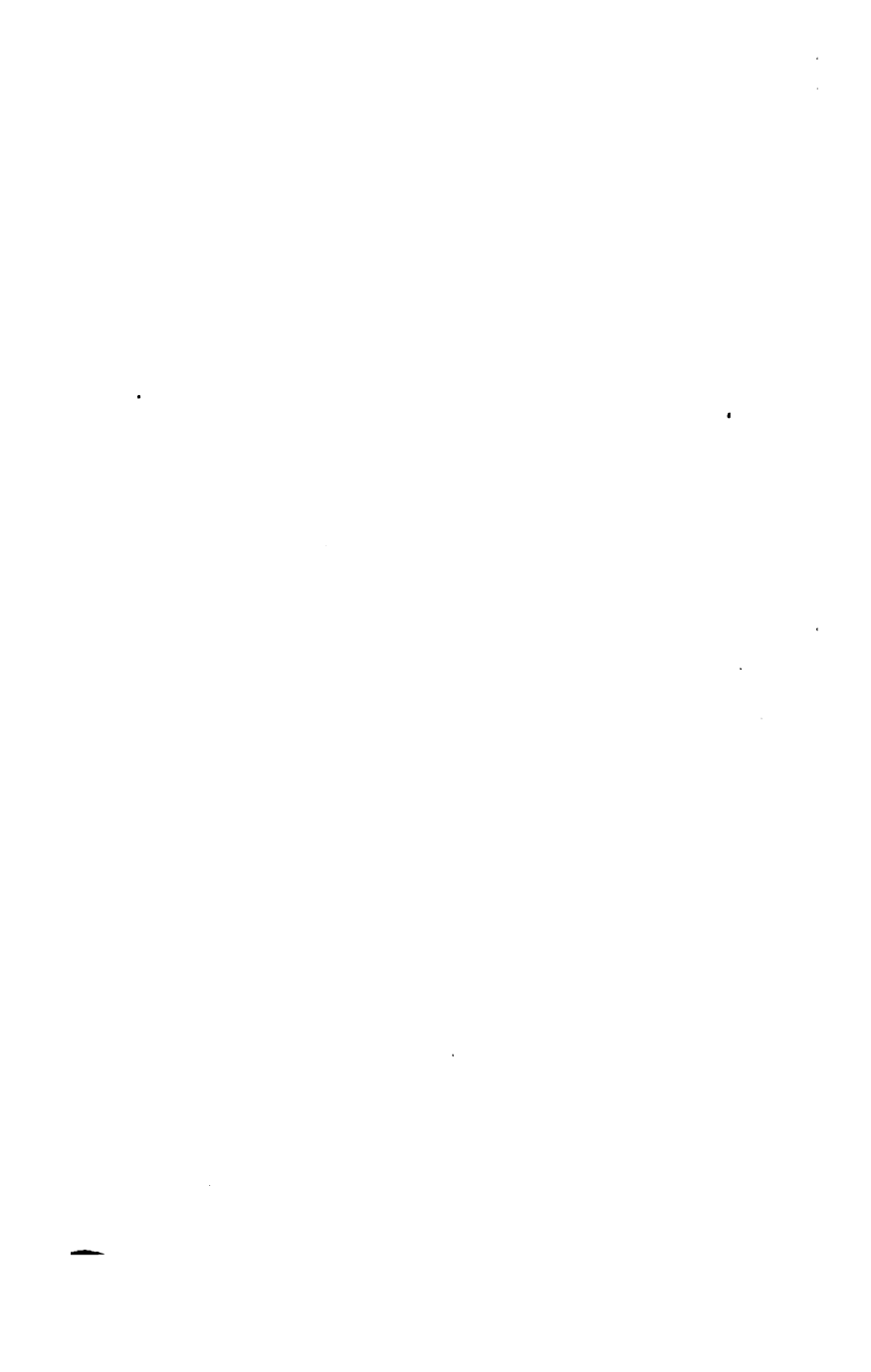
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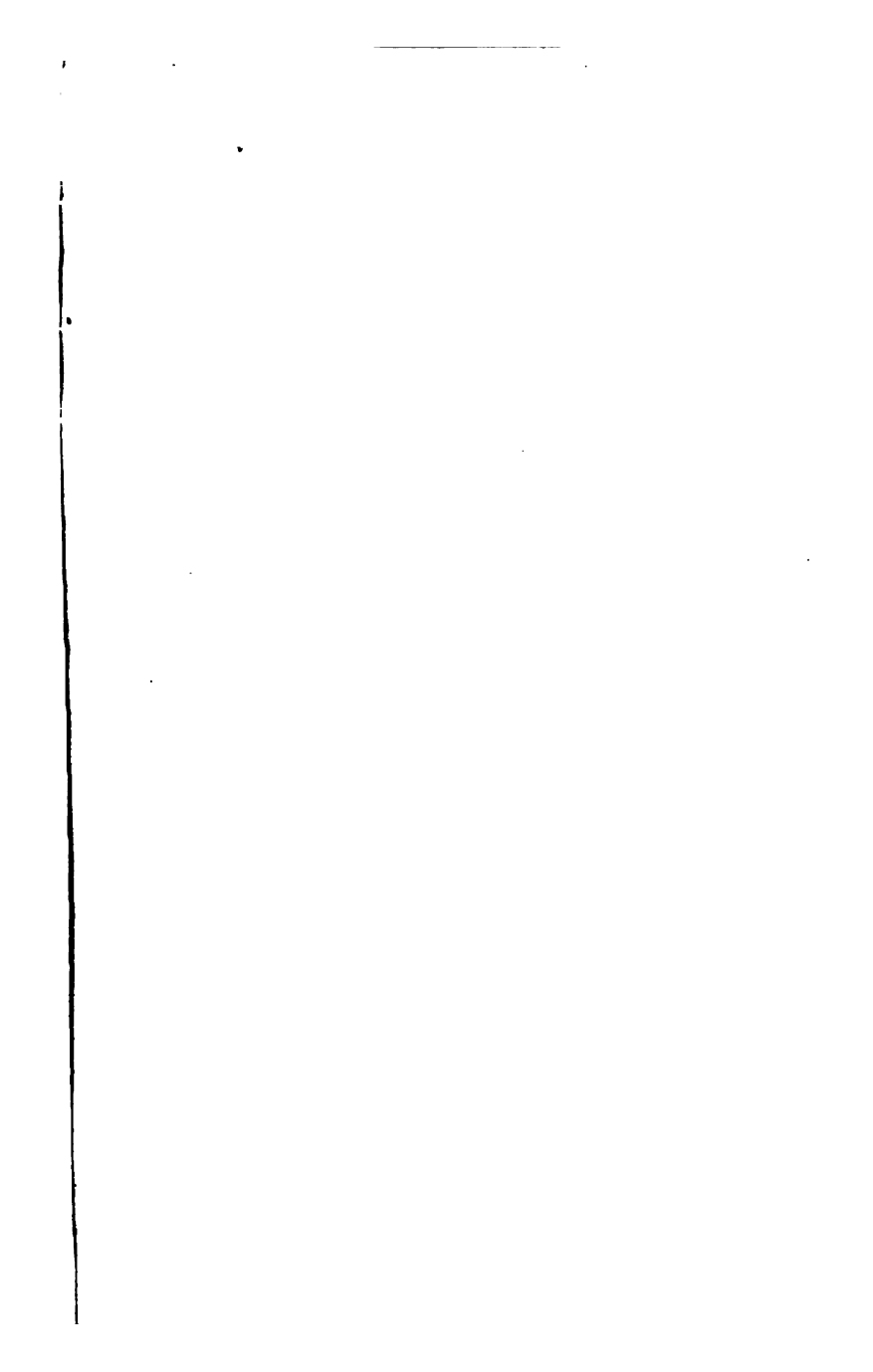
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THE
ELEMENTS
OF
EUCLID
EXPLAIN'D

In a *New*, but most *Easie* Method :

Together with

The U s E of every *Proposition* through all Parts of
the MATHEMATICKS.

Written in *French* by that Excellent Mathematician,
F. CLAUD. FRANCIS MILLIET de CHALES,
of the *Society of JESUS*.

Now made *English*, and a Multitude of Errors Cor-
rected, which had escap'd in the Original.

The SEVENTH EDITION.

L O N D O N :

Printed for JAMES and JOHN KNAPTON, DA-
NIEL MIDWINTER, BENJ. MOTTE,
AARON WARD, THOMAS COMBES,
JOHN CLARK, and JAMES LACY. 1726.

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Professor William H. Butler
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THE PREFACE.

HAVING for a long time observ'd, that most of those that take in hand the Elements of Euclid, are apt to dislike them, because they cannot presently discern to what end those seemingly inconsiderable, and yet difficult Propositions, can conduce: I thought I should do an acceptable piece of Service, in not only rendering them as easie as possible, but also adding to each Proposition a brief Account of some Use that is made of them in the other Parts of the Mathematicks. In prosecuting which Design, I have been oblig'd to change some Demonstrations, that seem'd too intricate and perplex'd, and above the ordinary Capacity of Beginners, and to substitute others more intelligible in their stead. For the same reason, I have demonstrated the Fifth Book after a Method much more clear than that by Equimultiples, formerly used. I would not be thought to have set down all the

P R E F A C E.

*Uses that may be made of these Propositions : To have done that, would have obliged me to have compriz'd the whole Body of Mathematicks in this one Book ; which would have render'd it both too large, and too difficult. But I have contented my self with the choice of such as may serve to point out some of the Advantages they afford us, and are also in themselves most clear, and most easie to be apprehended. I have distinguish'd them by * Inverted Comma's, that the Reader may know them, not desiring he should dwell too long upon them, or labour to understand them perfectly at first, since they depend on the Principles of the other Parts.*

This therefore being the Design of this small Treatise, I voluntarily offer it to the Publick, in an Age whose Genius seems more addic'ted to the Mathematicks, than any that has preceded it.

* Instead of the Author's *Italian* Character.

EIGHT BOOKS
OF THE
Elements of *EUCLID*.

Together with
The Use of the *Propositions*.

THE FIRST BOOK.

THE Design of *EUCLID* in this Book, is to lay down the First Principles of *Geometry*; and, to do it methodically, he begins with *Definitions*, and the Explication of the most ordinary Terms. To these he adds some *Postulata*: And then proposing those known *Maxims* in which Natural Reason does instruct us; he pretends not to advance a step farther without a *Demonstration*, but to convince every Man, even the most obstinate, that will grant nothing, but what is extorted from him. In the first Propositions, he treats of *Lines*, and the different *Angles*, which are form'd by their Concourse; and having occasion to compare divers *Triangles* together, in order to demonstrate the Properties of *Angles*, he makes that

A 3

the

‘ the Business of the eight first Propositions.
 ‘ Then follow some Practical Instructions, how
 ‘ to divide an *Angle* and a *Line* into two Parts,
 ‘ and to draw a *Perpendicular*. Next, he shews
 ‘ the Properties of a *Triangle*, together with
 ‘ those of *Parallel Lines*; and having thus fi-
 ‘ nish’d the Explication of this Figure, he passes
 ‘ on to the *Parallelograms*, teaching the man-
 ‘ ner of reducing any *Polygon*, Multangular Fi-
 ‘ gure, into one more regular. Lastly, He fi-
 ‘ nishes the First Book with that famous Propo-
 ‘ sition of Pythagoras, *That in every Rectangular*
*Triangle the Square of the * Base is equal to the*
Square of both the other Sides.

* He calls that the *Base*, which is commonly call’d the
Hypotenuse, i. e. the Line that is opposite to the right Angle.

DEFINITIONS.

1. **A** *Point is that which hath no Parts.*

‘ This Definition must be understood in this
 ‘ Sense: *That Quantity*, which we conceive
 ‘ without distinguishing its Parts, or so much
 ‘ as considering whether or no it has any, is a
 ‘ *Mathematical Point*; which is therefore very
 ‘ different from those of *Zeno*, which were sup-
 ‘ pos’d to be absolutely indivisible, and there-
 ‘ fore such, that we may reasonably doubt whe-
 ‘ ther they are possible; but the former we can-
 ‘ not doubt of, if we conceive them aright.

2. *A Line is Length without Breadth.*

‘ The Sense of this Definition is the same with the former: *That* Quantity, which we conceive as Length, without reflecting on its Breadth or Thickness, is that which we understand by a Line; though it be impossible to draw a real Line, which will not be of a certain Breadth. ’Tis commonly said that a Line is produc’d by the Motion of a Point: Which ought to be carefully observ’d, for Motion may on that manner produce any Quantity whatsoever: But here, we must imagine a Point to be only so mov’d, as to leave one Trace in the Space through which it passes, and then that Trace will be a Line.

3. *The two Extremes of a Line are Points.*

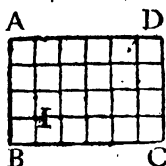
4. *A right Line is that, whose Points are equally plac’d between the two Extremes.*

‘ Or thus. A right Line is the shortest that can be drawn from one Point to another. Or yet; the Extremes of a right Line may cast a Shadow upon the whole Line.

5. *A Superficies, or Surface, is a Quantity to which is attributed Length and Breadth, without the Consideration of any Thickness.*

6. *The Extremes of a Superficies are Lines.*

7. *A Plain or right Superficies, is that, whose Lines are equally plac’d between its two Extremes; Or that, to which a right Line may be every way apply’d.*



‘ I have before observ’d, That
 ‘ Motion may produce any
 ‘ Quantity whatsoever: Accord-
 ‘ ingly we say, when one Line
 ‘ moves over another, it produ-
 ‘ ces a Superficies, or a Plane;
 ‘ and that *that* Motion has a kind of Affinity
 ‘ with *Arithmetical* Multiplication, Suppose
 ‘ then the Line A B to pass along the Line B C,
 ‘ retaining still the same Situation, without any
 ‘ Inclination to one side or other: The Point A
 ‘ will describe the Line A D, the Point B the
 ‘ Line B C, and the intermediate Points the
 ‘ Lines parallel to those, which will make up
 ‘ the Superficies ABCD. I add further, That
 ‘ this Motion answers to *Arithmetical* Multi-
 ‘ plication: Because, did I know the Number
 ‘ of Points that are contain’d in both those
 ‘ Lines, A B, and D C; by multiplying them
 ‘ together, I should find a Product, which would
 ‘ give me the Number of Points which consti-
 ‘ tute the whole Superficies ABCD. As for
 ‘ Example; If A B contain’d four Points, and
 ‘ B C six, by saying, four times six make Twenty-
 ‘ four, I find, that the whole Superficies ABCD
 ‘ consists of Twenty-four Points. Now by a
 ‘ *Mathematical* Point, may be understood any
 ‘ Quantity whatsoever; *e. g.* a Foot, provided
 ‘ it be not subdivided into Parts.

8. *A Plain Angle, is the * Distance or opening of two Lines touching each other, so as not to compose only one Line.*

9. As

* Overture. Gall. πρὸς ἀλλήλους κλίσις. Eucl.

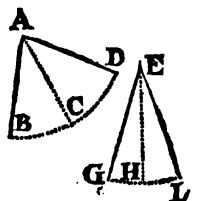
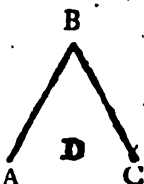
9. As the Distance D betwixt the Lines AB, and BC; which are not Parts of the same Line.

A Rectilineal Angle is the Distance betwixt two Right Lines.

'Tis chiefly of this sort of Angles that I would be understood at present; which I define by *Distance or Opening*, because Experience teaches, that the greatest part of Beginners deceive themselves in Measuring the greatness of an Angle, by that of the Lines within which it is contain'd.

The Angle that is more open, is the greater; that is, when the Lines of one Angle lie more apart from each other than those of another, taking them at the same Distance from the Points of Concourse, the former is greater than the latter. Accordingly the Angle A is greater, than the Angle E; because taking the Points D and B as remote from the Point A, as the Points G and L are from the Point E; the Points D and B lie farther apart from each other, than the Points G and L. From whence I infer, that if the Lines EG and EL were produc'd farther, the Angle E would be always of the same Largeness, and always less than the Angle A.

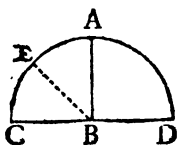
We use three Letters when we speak of an Angle, of which the Middlemost denotes the Point



‘ Point of Concourse : As the Angle BAD is the
 ‘ Angle which by the Lines BA and AD, is
 ‘ form’d at the Point A: The Angle BAC is that
 ‘ made by the Lines BA and AC: The Angle
 ‘ CAD is compris’d by the Lines CA and AD.

‘ A Circle is the Measure of an Angle. There-
 ‘ fore to know the Magnitude of the Angle
 ‘ BAD, I place the Foot of the Compass upon
 ‘ the Point A, and describe the Circle BCD:
 ‘ The Angle is so much the greater, by how ma-
 ‘ ny more Parts of a Circle the Arch, that mea-
 ‘ sures it, contains: And because a Circle is u-
 ‘ sually divided into 360 Parts or Degrees, there-
 ‘ fore an Angle is said to have Twenty, Thir-
 ‘ ty, Forty Degrees, according as the Arch,
 ‘ compris’d betwixt the Lines that form it, con-
 ‘ tains so many. So the Angle is the greater,
 ‘ which contains more Degrees, as the Angle
 ‘ BAD is greater than the Angle GEL. The
 ‘ Line CA divides the Angle BAD in the mid-
 ‘ dle, because the Arches BC and CD are equal;
 ‘ and the Angle BAC is part of the Angle BAD,
 ‘ because the Arch BC is part of the Arch BD.

*10. When one Line falling upon another makes
 two equal Angles, they are both right Angles; and
 the Line Perpendicular.*



‘ As for Example: If the Line
 ‘ AB, placed upon the Line CD,
 ‘ makes the Angles ABC and
 ‘ ABD equal; that is, if ha-
 ‘ ving describ’d a Semi-circle
 ‘ CAD

‘ CAD from the Centre B, the Arches AC and
 ‘ AD are equal: The Angles ABC and ABD are
 ‘ called Right Angles, and the Line AB Per-
 ‘ pendicular. Therefore because the Arch
 ‘ CAD is a Semi-circle, the Arches CA and AD
 ‘ are each of them a quarter of a Circle, that
 ‘ is the fourth Part of three hundred and sixty
 ‘ Degrees, that is Ninety.

11. *An Obtuse Angle is that which is greater than a Right one.*

‘ As the Angle EBD is an Obtuse or Blunt
 ‘ Angle; because its Arch EAD contains more
 ‘ than a quarter of a Circle.

12. *An Acute Angle is that which is less than a right one.*

‘ As the Angle EBC is an Acute, because the
 ‘ Arch EC, which measures it, has less than
 ‘ ninety Degrees.

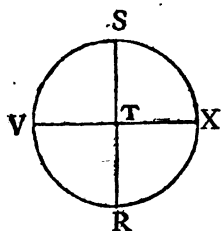
13. *A Term is the extremity or end of any Quantity.*

14. *A Figure is a Quantity comprehended by one or more Terms.*

‘ That which is call’d a Figure, ought to be
 ‘ limited and inclos’d on every Side.

15. *A Circle is a Plain Figure, terminated by the encompassing of one Line, which is call’d the Circumference; and is every where equally remote from the middle Point.*

‘ The



‘ The Figure RVSX is a
 ‘ Circle; because all the
 ‘ Lines TR, TV, TS, TX,
 ‘ drawn from the Point T,
 ‘ to the Line RVSX, are e-
 ‘ qual.

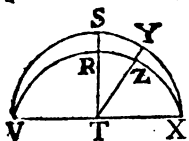
16. The middle Point is
 call'd the Centre.

17. The Diameter of a Circle, is any Line pas-
 sing through the Centre, and terminated at the Cir-
 cumference, dividing the Circle into two equal Parts.

‘ As the Lines V T X, and R T S:

‘ But if any should doubt, whether the Line
 ‘ VTX does indeed divide the Circle into two
 ‘ equal Parts, so that the Part VSX be equal to
 ‘ the Part VRX; it may on this manner be
 ‘ prov'd.

‘ Suppose the Part VRX to be placed upon
 ‘ the other VSX: I say, they will not exceed



‘ one the other. For if one sup-
 ‘ pose VSX exceed the other
 ‘ VRX, the Line TR will be
 ‘ less than TS; and in like man-
 ‘ ner TZ than TY; which is
 ‘ contrary to the Definition of a Circle, which
 ‘ affirms all the Lines drawn from the Centre to
 ‘ the Circumference to be equal.

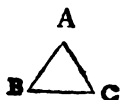
18. A Semi-circle is a Figure terminated by the
 Diameter, and half the Circumference.

19. Rectilineal Figures are such as are termina-
 ted by right Lines, having Three, or Four, or Five,
 or as many Sides as you please

‘ Euclid

‘Euclid divides Triangles with respect either
‘to their Angles, or Sides.

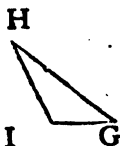
20. An Equilateral Triangle is
that which has its three Sides equal,
ABC.



21. An Ifofceles, or Equicrural Tri-
angle, is that which has two Sides e-
qual: ‘As if the two Sides AB and
‘AC be equal, the Triangle ABC
‘is an Ifofceles.

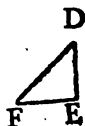


22. A Scalenum is a Triangle H
having all the three Sides unequal, as
GHI.

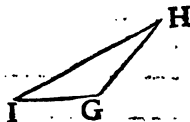


23. A Right-angle Triangle is that
which has one Right-Angle.

‘As DEF, fupposing the Angle E
‘to be a right One.



24. An Ambligone, or Ob-
tufe-angle Triangle, is that
which has one Angle Obtufe;
as IGH.

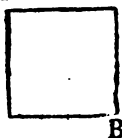


25. An Oxygone, or Acute-angle
Triangle, is that whose Angles are
all Acute; as ABC.



The Elements of Euclid.

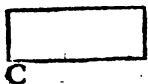
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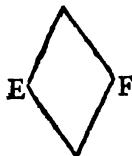
26. A Rectangle (properly so call'd) is a Figure consisting of four Sides, and having all its Angles Right.

27. A Square has all its Sides Equal, and its Angles Right; as AB.

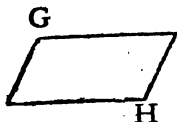
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28. An Oblong Rectangle has its Sides unequal, but its Angles Right; as CD.

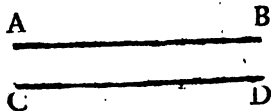


29. A Rhombus, or Losange, has equal Sides, but unequal Angles; as EF.

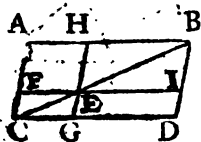


30. A Rhomboides, or Oblong Losange, hath both its Sides and Angles unequal; as GH.

31. Other Irregular Figures of Four Sides are call'd Trapezia.



32. Parallel Lines are such, as being in the same Plane, will never concur keeping still an equal Distance one from the other; AB, CD.



33. A Parallelogram is a Figure, whose two opposite Sides are Parallels: As the Figure ABCD, whose Sides, AB, CD; and AC, BD, are Parallels.

34. *The Diameter of a Parallelogram is a right Line drawn from one Angle to another; as BC.*

35. *The Complements are the two small Parallelograms, through which the Diameter does not pass; as AFEH, and GDIE.*

DEMANDS, or SUPPOSITIONS.

1. **T**HIS suppos'd that a right Line may be drawn from any Point whatsoever to another.

2. That a Right Line may be continu'd to what Length you please.

3. That from a Centre given, a Circle may be describ'd at any Distance whatsoever.

MAXIMS, or AXIOMS.

1. **T**Hose Quantities that are equal to a third, are equal betwixt themselves.

2. If equal Quantities be added to those that are equal, the Products will also be equal.

3. If equal Quantities be taken away from those that are equal, the Remainders will be equal.

4. If you add equal Parts to Quantities unequal, they will remain unequal.

5. If from equal Quantities you take away unequal Parts, the Remainders will be unequal.

6. Quantities that are Double, Triple, Quadruple, &c. in respect of the same, are equal among themselves.

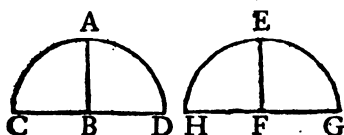
7. Those

7. Those Quantities are said to be equal, which being apply'd one to the other, neither exceeds.

8. Equal Lines and Angles being plac'd one upon another, do not surpass each other.

9. The Whole is greater than its Part.

10. All right Angles are equal to one another.



Let the two right Angles propos'd be ABC , EFH , I say, they are equal. For if two equal Circles CAD , HEG , be describ'd from the Centres B and F ; the fourth Parts of those Circles CA , HE , which are the Measures of the Angles, ABC , EFH , will be equal: Therefore the Angles ABC , EFG , having equal Measures, will be equal.



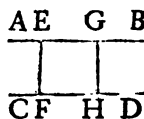
The eleventh Maxim of *Euclid* is to this Effect. If two Lines AB , CD , being cut by a third EF , make the Internal Angles, BEF , DFE , less than two right Angles; the Lines AB , CD being produced, will at length concur towards the Points B and D .

Which, though it be true, is not clear enough to be receiv'd for a Maxim; therefore I have substituted another in its Place.

11. If two Lines be parallel, all the Perpendiculars contained betwixt them will be equal.

As,

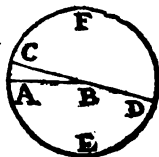
As for Examples : If the Lines
 ' AB, CD, are parallel, the Perpen-
 ' dicular Lines FE, HG, are equal.
 ' For if EF were greater than GH,
 ' the Lines AB and CD would be
 ' more remote from each other towards the
 ' Points E and F, than towards G and H; which
 ' would be contrary to the Definition of Paral-
 ' lels, where, 'tis said, they are such as always
 ' keep the same Distance, measur'd by Perpen-
 ' diculars.



12. Two Right Lines cannot enclose any Space; that is to say, they cannot encompass it on all Sides.

13. Two Right Lines cannot have one common Segment.

' By which I mean, That two
 ' Right Lines, suppose AB, and
 ' CB, meeting at the Point B,
 ' cannot together make one sole
 ' Line BD; but cutting one ano-
 ' other, separate again immedi-
 ' ately after their Rencontre. For if you de-
 ' scribe a Circle from the Point B as a Centre,
 ' AFD will be a Semi-circle; because the Right
 ' Line ABD, passing through the Centre B,
 ' will divide the Circle into two equal Parts.
 ' The Segment CFD will be also a Semi-circle;
 ' because CBD will be also a Right Line, and
 ' will pass through the Centre B: Therefore
 ' the Segment CFD will be equal to the Seg-
 ' ment AFD, the Part to the Whole; which
 ' is repugnant to the ninth Maxim.



ADVERTISEMENT.

‘ **T** Here are two sorts of Propositions. In
 ‘ some we have nothing but the bare Spec-
 ‘ culation of a Truth, without descending to Pra-
 ‘ ctice, which we call *Theorems*; in others some-
 ‘ thing is propos’d to be done, and those are
 ‘ call’d *Problems*.

‘ The first Number of the Quotations denotes
 ‘ the Propositions, the second the Book. As *by*
 ‘ *the 2. of the 3.* that is, by the second Proposi-
 ‘ tion of the third Book: But if only one Num-
 ‘ ber occur, it signifies such a Proposition of the
 ‘ Book you are then upon.

PROPOSITION I.

A PROBLEM.

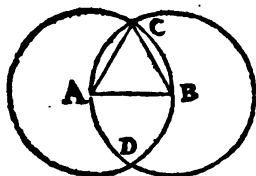
To draw an Equilateral Triangle upon any Line given-

L E T the Line AB be propos’d for the Base
 of an Equilateral Triangle; from the Centre A at the Distance AB describe the Circle BCD; and likewise from the Centre B at the Distance BA describe the Circle DAC, cutting the former at the Point C. Then draw the Lines AC and BC, and all the Sides of the Triangle ABC will be equal.

Demon-

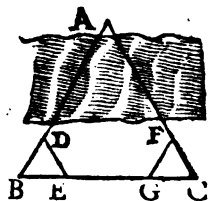
Demonstration.

The Lines AB and AC, being drawn from the same Centre A to the Circumference of the Circle BCD, are equal, *by the Definition of a Circle*; the Lines BA and BC are likewise equal, being drawn from the Centre B to the Circumference of the Circle CAD. Lastly, The Lines AC and BC being equal to the same Line AB, are also equal between themselves. All the three Sides therefore of the Triangle ABC are equal.



The USE.

‘ The Design of *Euclid* in
 ‘ placing this Problem here,
 ‘ was only to demonstrate the
 ‘ two following Propositions.
 ‘ But it may be also further
 ‘ serviceable for the measuring
 ‘ an inaccessible Line; as for Example, the Line
 ‘ AB, which by reason of a River or Precipice
 ‘ cannot be approach’d. In such a Case make
 ‘ a small Equilateral Triangle BDE, either of
 ‘ Wood or Copper, or the like; and having
 ‘ placed it horizontally upon B, observe the
 ‘ Point A, by the Side BD, and any other Point
 ‘ C, by the Side BE. Then transfer your Tri-
 ‘ angle along the Line BC, and place it upon
 ‘ divers parts of the same Line; till at length
 ‘ you find a Point C, upon which placing the
 ‘ Triangle

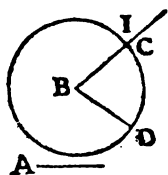


' Triangle you shall see the Point B, by the Side
 ' CG, and the Point A by the Side CF. I say,
 ' The Lines CB and CA (or AC) are equal; so
 ' that by measuring the Line BC, you may know
 ' the Line AB. I might further demonstrate,
 ' that the Lines AB and BC are equal; but let
 ' it suffice, that in this Proposition, you are
 ' taught the way of making an Instrument pro-
 ' per to take the Dimensions of an inaccessible
 ' Line.

PROPOSITION II.

A PROBLEM.

From a Point given, to draw a Line equal to another Line given.



LET the Point propos'd be
 B, from which a Line is to
 be drawn equal to the Line A.
 Take with the Compass the
 Length of the Line A, and at that
 Interval, making B the Centre,
 describe the Circle CD. Drawing then from the
 Point B to which Side you please, a Line BI or
 BD, 'tis evident it will be equal to the Line A.

' *Euclid* proposes a more mysterious and in-
 ' tricate Method of demonstrating this Proposi-
 ' tion; but in Practice we always make use of
 ' this: Inasmuch as, having taken with the Com-
 ' pass the Line A, 'tis as easie describing a Circle
 ' from the Centre B, as from the Centre A.

P R O.

PROPOSITION III.

A PROBLEM.

From a Greater Line to take a Part equal to a Less.

SUPPOSE you were to take from the Line BC a Part BI, equal to the Line A. Take betwixt the Points of the Compass the Length of the Line A, and at the Distance from the Centre B describe a Circle, which shall cut the Line BC at the Point I. 'Tis certain the Lines BI, and A, are equal.

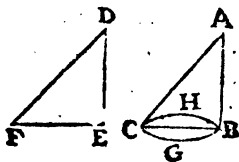
' The Use of these two preceding Propositions
' is sufficiently evident; forasmuch as we are fre-
' quently oblig'd in *Practical Geometry* to draw
' one Line equal to another, and to take a Part
' of a greater Line equal to a Line that is less.

PROPOSITION IV.

A THEOREM.

If two Triangles have two Sides equal, each to the other respectively, and the Angles also, form'd by those two Sides equal; their Bases and other Angles will be equal.

LET the Triangles ABC, DEF, have two Sides equal each to the other respectively; that is to say, Let AB be equal to DE, and AC to DF; and let



B 3

the

the Angles BAC, EDF, form'd by those Sides, be also equal: I say, the Bases BC, EF are equal; and the Angles ABC, DEF; ACB, DEF, are equal; and lastly, the whole Triangles are equal in all respects.

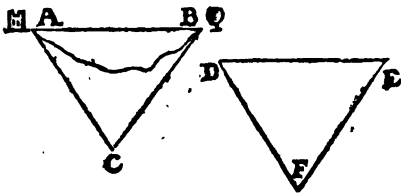
Demonstration.

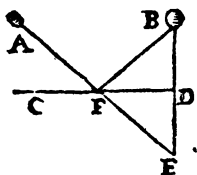
Suppose the Triangle DEF to be plac'd upon the Triangle ABC; the Side DE being upon AB, they will not exceed each other, because they are suppos'd to be equal; so that the Point E will be upon B, and the Point D upon the Point A. For the same Reason the Line DF will fall upon AC. For if it should fall on the Outside of it, the Angle EDF would be greater than the Angle BAC; and if it should fall within AC, the Angle EDF would be less; and yet they are suppos'd to be equal. Therefore since the Point D is upon the Point A, and the Line DF falls upon the Line AC, to which it is equal, they will not exceed each other, but the Point F will fall upon C. Lastly, Since the Points E and F of the Line EF, fall upon B and C; the Line EF will fall upon BC; because it can neither fall higher, as in BHC, nor lower, as in BGC; for then two Right Lines would enclose Space; which is contrary to the twelfth Maxim. Therefore the two Triangles do not at all exceed each other. But not only the Bases BC, EF, but also the Angles ABC, DEF, and ACB, and DFE, are equal.

Coroll.

Coroll. An Equilateral Triangle hath all its Angles equal.

The USE

‘ Suppose I  were to measure an inaccessible Line A B. I observe from the Point C, the Points A and B; and then measure the Angle C. This done, placing a Board horizontally, and observing successively by a Rule the Points A and B, I draw two Lines according to the Rule, which make the Angle C; and measure with a Yard the Lines AC, and BC, which are supposed accessible. Then going into an open Field, and placing my Board again horizontally upon the Point F, and observing the Lines that I drew upon it, make an Angle DFE equal to the Angle C. I make likewise FD, FE, equal to CA, CB. Then according to this Proposition, the Lines BB, and DE, are equal. So that measuring by the Yard the accessible Line DE, I shall know AB, which is inaccessible.



' The same Proposition may
 ' serve to teach how to hit a
 ' Bowl at Billiards by Reflec-
 ' tion. Suppose one Bowl to
 ' be at the Point A, and that
 ' which you would hit at the
 ' Point B, and CD the Billiard-Table. Imagine
 ' then a Perpendicular BDE, and take the Line
 ' DE equal to BD. I say, If you direct the Bowl
 ' from the Point A to E, the Reflection will car-
 ' ry it to B. For in the Triangles BFD, EFD,
 ' the Side FD being common, and the Sides BD
 ' and DE equal; as also the Angles BDF and
 ' EDF Right Angles, the Angles BFD, EFD are
 ' equal, by this Proposition. The Angles AFC
 ' DFB, being opposite, are also equal, as I
 ' shall demonstrate hereafter. Therefore the
 ' Angle of Incidence AFC, is equal to the Angle
 ' of Reflection BFD; and by consequence the
 ' Reflection will be by AFB.

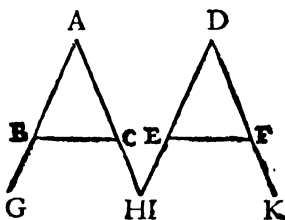
P R O-

PROPOSITION V.

A THEOREM.

In Iſoſceles, or Equicrural Triangles, the Angles that are above the Baſe are equal; as alſo thoſe that are below it.

LET the *Iſoſceles* be ABC , that is to ſay, let the Sides AB and AC be equal. I ſay, the Angles ABC , ACB are equal; as alſo the Angles GBC , HCB , that are below the Baſe BC . Suppose another Triangle DEF , having the Angle D equal to the Angle A ; and the Sides DE , DF equal to AB , AC . Since the Sides AB , AC are equal, all the four Lines AB , AC , DE , DF will be equal.



Demonſtration.

Since the Sides AB , DE ; AC , DF , are equal; as alſo the Angles A and D ; if the Triangle DEF be plac'd upon ABC , they will not exceed each other, but the Line DE will fall upon AB ; DF upon AC ; and EF upon BC (by the 4th.) therefore the Angle DEF will be equal to ABC . And becauſe one part of the Line DE falls upon AB , the whole Line DI will be upon AG ; otherwiſe two Right Lines would have a common Segment; therefore the Angle IEF will be equal

equal to GBC. Suppose then the Triangle DEF turn'd, and apply'd another way to the Triangle ABC; that is to say, so as DF may fall upon AB, and DE upon AC. Since the four Lines AB, DF, AC, DE, are equal; as also the Angles A and D: The Triangles will likewise agree this way, and the Triangles ACB, DEF, HCB, IEF, will be equal. Now, by the comparing them, it appear'd, that the Angle ABC was equal to the Angle DEF; and GBC to IEF: Therefore the Angles ABC, ACB, being equal to the same DEF; and GBC, HCB, also equal to the same IEF, they are equal among themselves.

‘ I was unwilling to make use of *Euclid's*
 ‘ Demonstration; because being very difficult,
 ‘ it might discourage Beginners.

PROPOSITION VI.

A THEOREM.

If two Angles of a Triangle be equal, the Triangle will be an Isosceles.

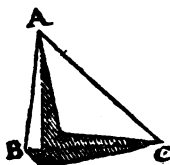
LET the Angles ABC, ACB of the Triangle ABC be equal: (*see Fig. preced.*) I say, it is an *Isosceles*; that is to say, the two Sides AB, AC, which are opposite to the equal Angles, are equal. Suppose the Triangle DEF to have a Base EF equal to BC, and the Angle DEF equal to ABC, as also DFE equal to ACB: since the Angles ABC, ACB are suppos'd to be equal, all
 the

the four Angles ABC , ACB , DEF , DFE , will be equal. Suppose again therefore the Base EF to be plac'd upon the Base CB , so that the Point E lie upon the Point B , the Bases being suppos'd equal, it is evident they will not exceed each other. Further, the Angle E being equal to the Angle B , and the Angle F to the Angle C ; the Line ED will fall upon the Line BA , and FD upon CA : So that the Lines ED and FD will meet at the Point A . From whence it follows, that the Line ED is equal to BA .

Let then the Triangle DEF be turn'd to the other Side, and applied another way to the Triangle ABC ; that is to say, so that the Point E lie upon C , and F upon B : the Bases BC , FE will perfectly agree, being suppos'd to be equal: And because the Angles F , and B , E , and C , are also suppos'd to be equal, the Side FD will fall upon BA , and ED upon CA ; and the Point D , upon A . Therefore the Lines AC , DE will be equal. Whence it follows, that the Sides AC , AB are equal between themselves, being equal to the same Side DE .

The U S E.

- ' This Proposition may serve
- ' for taking the Dimensions of
- ' any sort of inaccessible Lines.
- ' 'Tis said that *Thales* was the
- ' first that measur'd the Height
- ' of Obelisks by their Shadows:
- ' It may be done by this Proposition. For if
- ' you



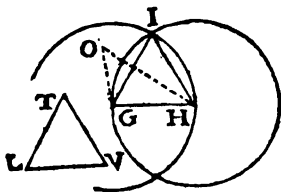
' you were to measure the Height of the Obelisk AB; do but expect 'till the Sun be elevated 45 Degrees above the Horizon; that is to say, 'till the Angle ACB be 45 Degrees; and, by this sixth Proposition, the Shadow BC will be equal to the Obelisk AB. For since the Angle ABC is a right Angle, and the Angle ACB half a right one, or of 45 Degrees; the Angle CAB will be half a Right one, as I shall prove hereafter. Therefore the Angle BCA, BAC, are equal: and (by the 6.) the Sides AB, BC, are also equal. I can also measure the same Height without making Use of the Shadow, by taking a Stand so far from the Point B, as that the Angle ACB may be half a Right Angle; which may be known by a Quadrant. These Propositions are of frequent Use in Trigonometry, and in all other Tracts. The Seventh Proposition may be omitted, because 'tis of no other Use but to demonstrate the Eighth, which may be done without it.

PROPOSITION VIII.

A THEOREM.

If two Triangles have all their Sides equal, their opposite Angles will also be equal

LET the Side GI be equal to LT; HI, to VT, GH, to LV; I say, That the Angle GIH will be equal to the Angle LTV; IGH, to the



Angle L; and IHG to the Angle V. From the Centre H, at the Distance HI, describe the Circle IG; and from the Centre G, at the Distance GI, the Circle HI.

Demonstration.

Suppose the Line LV brought upon HG; they would not exceed each other, because they are suppos'd to be equal. I add, That the Point T will fall precisely upon the Point I: For it ought to reach precisely to the Circumference of the Circle IG, because by the Supposition the Lines HI and VT are equal. It ought in like manner to reach to the Circumference of the Circle IH, because the Lines GI and LT are equal. So then it will light upon the Point I, being the Point where those two Circles cut each other. Indeed, if it fell any where else, as upon O, the Line HO, that is to say, VT, would be

be greater than HI; and the Line GO; that is, LT, would be less than GI; which is against the Supposition. Whence I conclude, That the Triangles will exactly correspond, and the Angle GIH be equal to the Angle LTV.

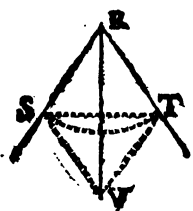
The USE.

‘ This Proposition is necessary for the Proof
 ‘ of those that follow. And further, When we
 ‘ cannot take the Measure of an Angle, because,
 ‘ the Lines meeting in a Solid Body, we cannot
 ‘ apply our Instruments to it; we must take the
 ‘ three Sides of the Triangle, and make another
 ‘ upon a Paper, whose Angles we may measure.
 ‘ This is a very ordinary Practice in *Gnomonicks*,
 ‘ or *Dialling*; and in the Treatises concern-
 ‘ ing cutting Precious Stones, so as to fit the Pan-
 ‘ nels, and to retain the Waters.

PROPOSITION IX.

A PROBLEM.

To divide an Angle into two equal Parts.



LET the Angle of SRT be propos'd to be divided into two equal Parts. Take the Compass, and from the Centre R, at any Distance, draw the Arch ST, cutting off two equal Lines RS, RT. Then draw the Right Line ST, and (*by the 1.*) describe an Equilateral

Equilateral Triangle STV . I say, The Line VR divides the Angle into two equal Parts; that is to say, the Angles VRT , and VRS , are equal.

Demonstration.

The Triangles VRS , and VRT , have the Side VR common; and the Side RT was taken equal to the Side RS : The Base also SV is equal to VT , because the Triangle SVT is equilateral. Wherefore (*by the 8.*) the Angles SRV , VRT , are equal.

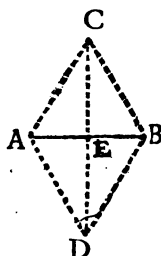
The USE.

‘ This Proposition is very useful to divide the
‘ Fourth part of a Circle into Degrees: For ’tis
‘ the same thing to divide an Arch as an Angle
‘ into two equal Parts; and the Line RV does
‘ both; that is, It divides both the Arches ST ,
‘ and the Angle SRT . Having therefore ap-
‘ ply’d the Semi-diameter to the fourth Part of
‘ a Circle, you cut off an Arch of 60 Degrees;
‘ which divided equally, gives an Arch of 30;
‘ and that again divided, makes one of 15 De-
‘ grees. ’Tis true, to finish this Division, we
‘ must divide an Arch into three equal Parts, but
‘ that is not to be done Geometrically. Pilots
‘ also divide the Compass into 32 Winds by the
‘ help of this Proposition only.

PROPOSITION. X.

A PROBLEM.

To divide a Right Line into two equal Parts.



Suppose the Line AB was to be divided into two equal Parts; upon the Line AB describe an Equilateral Triangle ABC, (*by the 1.*) and divide the Angle ACB into two equal Parts by the Line DC, (*by the 9.*) I say, the Line AB is divided equally at the Point E; that is, to say, the Lines AE and EB are equal.

Demonstration.

The Triangles ACE and BCE have the Side CE common, and the Sides CA and CB are equal, because the Triangle ACB is equilateral; and the Angle ACB being divided equally, the Angles ACE and BCE are also equal. Therefore (*by the 4.*) the Bases AE and BE are equal.

The USE.

‘Great Use is made of this Proposition, ordinary Practices frequently requiring us to divide a Line in the Middle, which *Geometricians* require should be done exactly at the first Dash, by a Method that is infallible, and not by Essays. This Practice is likewise principally useful for dividing Measures into less Parts.

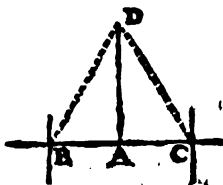
P R O-

PROPOSITION XI.

A PROBLEM.

To draw a Perpendicular to a Line given, upon a given Point of the same Line.

Suppose you were to raise a Perpendicular upon the Point A in the Line BC: Take two equal Lines, AB and AC, on both Sides the Point A, and make an Equilateral Triangle BDC upon the Line BC (by the 1.) I say, The Line AD is Perpendicular; that is to say, The Angles BAD and CAD are equal.



Demonstration.

The Triangles BAD and CAD have the Side AD common, the Sides AC and AB are equal, and the Bases BD and DC also equal: Therefore (by the 8.) the Angles BAD, and CAD, are equal; and (by the 10. defin.) the Line AD Perpendicular to BC.

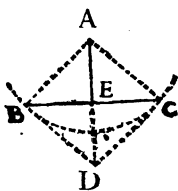
C

P R O-

PROPOSITION XII.

A PROBLEM.

To draw a Perpendicular to a Line given, from a Point which is out of the Line.



IF you would draw a Perpendicular to the Line BC from the Point A: Having set the foot of the Compass upon A, describe the Circle BC, which shall cut the Line BC, at the Points B and C. Then divide the Line BC into two equal Parts at the Point E. I say, the Line AE is Perpendicular to BC. Draw the Line AB, AC.

Demonstration.

The Triangles BEA, and CEA, have the Side AE common; and the Sides EC and EB equal, the Line BC having been equally divided at the Point E; the Bases AB and AC, being drawn from the Centre A to the Circumference BC, are likewise equal: Therefore the Angles AEB, and AEC are equal, (*by the 8.*) and the Line AE Perpendicular, (*by Defin. 10.*)

The Method in Practice, of dividing the Line BC in the Middle, is to describe two Arches at D, at the same Interval, from the Centres B and C.

The

The U S E.

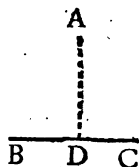
‘ We have need of a Plummert or Squaring
‘ line almost in all our Operations: No Angles
‘ are in Use in Building but the Right; and all
‘ Chairs, Benches, Tables, Buffets, and other
‘ Moveables, are framed by the Square. No Sur-
‘ vey of Land can be taken without making Use
‘ of Perpendicular Lines: Nor can *Dialling* be
‘ performed without them. The Carpenter’s
‘ Level contains a right Angle, and the same is
‘ preferr’d before any other, especially by the
‘ *French*, in Fortification. Lastly, Not only
‘ *Mathematicians*, but also the greatest part of
‘ practical Artisans, require that we should
‘ know how to draw a Perpendicular.

P R O P O S I T I O N XIII.

A T H E O R E M.

*One Line falling upon another, makes with it either
two right Angles or two Angles equal to two
Right ones.*

L E T the Line AD fall upon BC;
I say, ’twill make with it ei-
ther two right Angles, or two An-
gles, one Obtuse, and the other A-
cute, which join’d together, shall be
of equal Value with two Right ones.



Demonstration.

Suppose the Line AD to fall perpendicularly
upon BC, then ’tis evident (*by defn. 10.*) that

he Angles ADB, and ADC, are equal, and by consequence right Angles. Or,

Secondly, Suppose the Line ED not to fall perpendicularly upon BC, then having drawn a Perpendicular AD (*by the 11.*) the Angles ADB, and ADC, are right Angles, which are of equal Value with the three Angles ADC, ADE, EDB. But the Obtuse Angle EDC, and the acute Angle EDB, are of equal Value with the three Angles ADC, ADE, and EDB: Therefore the Angles EDC, and EDB, are of equal Value with two Right ones.

This Proposition may be more easily demonstrated, by describing a Semi-circle from the Centre D upon the Line BC. For the Angles EDB, and EDC, will require a Semi-circle for their Measure, which is the Measure of two right Angles, as I have shown before (*in the 8. Defin.*)

Corollary 1. If the Line AD falling upon BC, make one right Angle ADC, it is evident, the other ADB will be also a right Angle.

Coroll. 2. If the Line ED, falling upon BC, make the Angle EDB Acute; the Angle EDC will be Obtuse.

The USE.

‘ By this means, when we know one of the
 ‘ Angles which is made by one Line falling up-
 ‘ on another, we know also the other. As for
 ‘ Example:

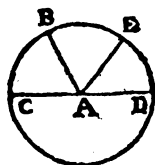
‘ Example: If the Angle EDB be one of 70
 ‘ Degrees, taking away 70 from 180, there will
 ‘ remain 110 for the Angle EDC. This Opera-
 ‘ tion does frequently occur in *Trigonometry*, and
 ‘ also in *Astronomy*, for finding the Excentricity
 ‘ of the Circle through which the Sun annually
 ‘ passes.

PROPOSITION XIV.

A THEOREM.

If two Lines meeting together at the same Point of another Line, make with it two Angles equal to two Right ones; they will make but one and the same Line.

Suppose the Lines CA. and DA to meet at the Point A of the Line AB; and that the Angles adjoining, CAB, and BAD, are equal to two Right ones. I say, the Lines CA and DA are but one and the same Line; so that CA being continued, will fall precisely upon AD.



Imagine, if you please, that CA continu'd, will pass on to E, and from the Centre A describe a Circle.

Demonstration.

If you say that CAE is a Right Line, the Arch CBE will be a Semi-circle. But 'tis suppos'd, that the Angles CAB, and BAD, are equal to

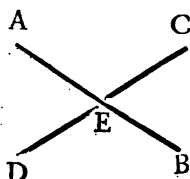
two Right ones, and that therefore their Measure is a Semi-circle. Therefore the Arches CBE and CBD will be equal; which is impossible, one being a part of the other. Therefore the Line CA being continu'd, will make but one and the same Line with AD.

PROPOSITION XV.

A THEOREM:

*If two Right Lines cut each other, the Opposite
* Angles at the Top will be equal.*

* κατὰ κορυφήν Eucl. au sommet. Gall.



LET the Line AB and CD cut each other at the Point E: I say, The Angles AEC, and DEB, which are opposite at the Top, are equal.

Demonstration.

The Line CE falling upon the Line AB, makes the Angles AEC and CEB equal to two Right ones, (*by the 13.*) In like manner the Line BE falling upon the Line CD, makes the Angle CEB and BED equal to two Right ones. Therefore the Angles AEC, CEB taken together, are equal to the Angles CEB, BED; therefore taking away the Angle CEB from both the Angle AEC, will remain equal to DEB (*by the 3. Maxim.*)

Coroll.

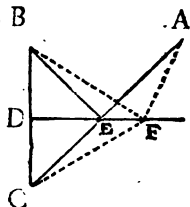
Coroll. If two Lines DE, and EC, concurring at the same Point E of the Line AB. form with it the opposite Angles AEC, DEB equal, DE and EC make but one Right Line.

Demonstration.

The Line EC falling upon the Line AB, makes the Angles AEC, and CEB equal to two Right ones, (*by the 13.*) 'Tis suppos'd likewise that the Angle DEB is equal to the Angle AEC. Therefore the Angles DEB, BEC, are equal to two Right ones. And (*by the 14.*) the Lines CE and ED make but one Right Line.

The US E.

' The two preceding Propo- B
' sitions are made Use of to
' prove, that two Lines make
' but one Total. As for Exam- D
' ple: In Catoptricks or Perspe-
' ctives, where that is required
' to prove, that of all the Lines C
' that can be drawn by Reflection from the Point
' A to the Point B, those are the shortest, which
' make the Angle of Incidence equal to the
' Angle of Reflection. As for Example: If the
' Angles BED and AEF be equal, the Lines AE,
' and EB, are shorter than AF, and FB. From
' the Point B draw a Perpendicular BD, and
' make the Lines BD and CD equal; then draw
' EC, and FC. First, In the Triangles BED
' and CED, the Side DE is common; and the



‘ Sides BD and BC being equal, as also the
 ‘ Angles BDE, and CDE; the Bases BE and
 ‘ CE will be equal; as also the Angles BED,
 ‘ and DEC, (*by the 4.*) In like manner, I may
 ‘ prove that BF and CF are equal.

Demonstration.

‘ The Angles BED and DEC are equal, and
 ‘ the Angles BED and AEF are suppos’d like-
 ‘ wise to be equal; therefore the opposite Angles
 ‘ DEC and AEF will be equal; and (*by the*
 ‘ *Coroll. of the 15.*) AEC one right Line; and
 ‘ by consequence AFC is a Triangle, of which
 ‘ the Sides AF and FC must be longer than
 ‘ AEC, that is to say, than AE, and EB. But
 ‘ the Lines AF and FC are equal to the Lines
 ‘ AF, and FB; therefore the Lines AF and FB
 ‘ are longer than the Lines AE and EB. And
 ‘ since Natural Causes always act by the
 ‘ shortest Lines, the Reflection will always hap-
 ‘ pen in such a manner, that the Angles of Re-
 ‘ flection and Incidence shall be equal.

‘ Further, Because we can easily prove, that
 ‘ all the Angles that can be made upon a Plane
 ‘ about the same Point, are equal to four right
 ‘ Angles, (forasmuch as in the first Figure of
 ‘ this Proposition, the Angles AEC and AED
 ‘ are equal to two Right ones, as also BEC and
 ‘ BED to two more) we make a general Rule
 ‘ to determine what Polygons may be join’d in
 ‘ paving a Hall. Accordingly we say, that Four
 ‘ Squares, Six Triangles, and three Hexagones,
 ‘ may

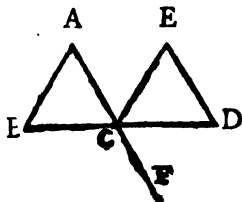
may be used for that Purpose: And that therefore Bees are always observ'd to make their little Cells of the last, that is, of Figures consisting of six Sides.

PROPOSITION XVI.

A THEOREM.

The External Angle of a Triangle is greater than either of the Internal Opposite Angles.

Produce the Side BC of Triangle ABC: I say, the External Angle ACD is greater than either of the Internal Opposite Angles, ABC, or BAC. Suppose the Triangle ABC to be mov'd along the Line BD, and carry'd into the Place of CED.



Demonstration.

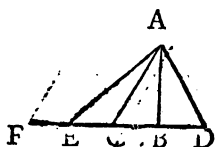
'Tis impossible that the Triangle ABC should be so mov'd, but the Point A must come into the place of the Point E; and then 'twill appear, that the Angle ECD; that is to say, ABC, is less than the Angle ACD: Therefore the Internal Angle ABC is less than the External ACD.

'Tis likewise easie to prove, that the Angle A is less than the External Angle ACD: For having prolong'd the Side AC as far as F, the
opposite

opposite Angles BCF, and ACD, are equal (*by the 15.*) Therefore causing the Triangle ABC to slide along the Line ACF, I shall demonstrate the Angle BCF to be greater than the Angle A.

The USE.

‘ We may draw from this Proposition many
‘ useful Conclusions. As first, That from a
‘ Point given, only one Perpendicular can be
‘ drawn to the same Line. For Example: Sup-



‘ pose the Line AB to be Per-
‘ pendicular to the Line BC;
‘ I say, That AC will not be
‘ Perpendicular; because the
‘ right Angle ABD must be
‘ greater than the Internal
‘ Angle ACB; therefore ACB cannot be a right
‘ Angle, nor AC a Perpendicular.

‘ Secondly, That from the same Point A can-
‘ not be drawn more than two equal Lines; for
‘ Example, AC, and AD; and if you draw a
‘ third as AE, it will not be equal to the former.
‘ For since AC and AD are equal, the Angles
‘ ACD, and ADC are equal, (*by the 5.*) but in
‘ the Triangle AEC, the External Angle ACB
‘ is greater than the Internal AEC: And there-
‘ fore likewise the Angle ADE is greater than
‘ the Angle AED. Therefore the Lines AE,
‘ and AD, are not equal; nor by consequence
‘ AC and AE.

‘ Thirdly, that if the Line AC makes the
‘ Angle ACB Acute, and ACF Obtuse, the
‘ Perpen-

‘ Perpendicular drawn from the Point A will
 ‘ fall on the side of the acute Angle. For if
 ‘ you say that AE is a Perpendicular, and
 ‘ that AEF is a Right Angle; the Right Angle
 ‘ AEF would be greater than the Obtuse ACE.
 ‘ These Conclusions are serviceable for measur-
 ‘ ing Parallelograms, Triangles, and Trapezia,
 ‘ and to reduce them into Rectangular Figures.

PROPOSITION XVII.

A THEOREM.

Any two Angles of a Triangle are less than two Right ones.

LET the Triangle be ABC; I say, That any two of its Angles taken together, as BAC, and BCA, are less than two Right ones. Produce the Side CA to the Point D.



Demonstration.

The Internal Angle C, is less than the External BAD, (*by the 16.*) Add therefore to both the Angle BAC; the Angles BAC, and BCA, will be less than the Angles BAC, and BAD; yet those are but equal to two Right ones, (*by the 13.*) Therefore the Angles BAC, and BCA, are less than two Right ones.

After

After the same manner I can demonstrate the Angles ABC, and ACB, to be less than two Right ones, by producing the Side BC.

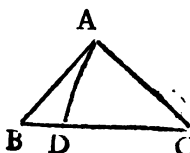
Coroll. If one Angle of a Triangle be a Right, or Obtuse Angle, the other are Acute.

‘ This Proposition is necessary to demonstrate those that follow.

PROPOSITION XVIII.

A THEOREM.

In every Triangle whatsoever, the greatest Side is oppos'd to the greatest Angle.



Suppose the Side BC of the Triangle ABC, to be greater than the Side AC; I say, The Angle BAC, that is oppos'd to the Side BC, is great than the Angle B, which is oppos'd to the Side AC. Cut the Line BC in D, so that CD may be equal to AC; then draw the Line AD.

Demonstration.

Since the Sides AC and CD, are equal, the Triangle ACD will be an *Isoceles*, and (by the 5.) the Angles CDA, and CAD, equal. Now the whole Angle BAC is greater than the Angle CAD: Therefore the Angle BAC is greater than the Angle CDA; which yet being an External Angle in respect of the Triangle ABD,

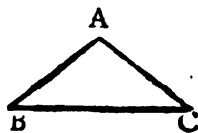
is greater than the Internal B, (*by the 16.*)
Therefore the Angle BAC is greater than the
Angle B.

PROPOSITION. XIX.

A THEOREM.

*In every Triangle, the greatest Angle is oppos'd to
the greatest Side.*

LET the Angle A of the
Triangle BAC be greater
than the Angle ABC. I say,
The Side BC, which is oppos'd
to the Angle A, is greater than
the Side AC, that is oppos'd to the Angle B.



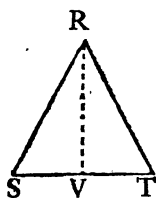
Demonstration.

If the Side BC be not greater than the Side AC, 'tis either equal; and then the Angles A and B would be equal; (*by the 5.*) which is contrary to the Supposition: Or less, and if so, the Side AC being greater than BC, the Angle BC would be greater than the Angle A, though the contrary be suppos'd. It remains therefore, that the Side BC is greater than the Side AC.

The USE.

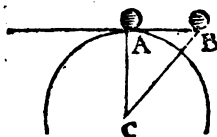
' We may prove from these Propositions, not
' only that no more than one Perpendicular
' can

‘ can be drawn from the same Point to the same
‘ Line; but also that it is the shortest of all.



‘ As for Example: If the Line
‘ RV be Perpendicular to ST, it
‘ will be less than RS; Because
‘ the Angle RVS being a right
‘ Angle, the Angle RSV will be
‘ an Acute, (*by the Coroll. of the*
‘ 17.) and the Line RV will be
‘ less than RS, (*by the preceding.*) Therefore
‘ Geometricians do always make use of a Per-
‘ pendicular, when they take the Dimensions
‘ of any Thing, and reduce irregular Figures
‘ to such as have one or more right Angles. I
‘ add, That it being impossible that more than
‘ three Perpendiculars should meet at the same
‘ Point, it cannot be imagin’d that there should
‘ be more than three Species or Kinds of Quan-
‘ tity, a Line, a Superficies, and a Solid
‘ Body.

‘ By these Propositions we likewise prove,
‘ That a Bowl exactly round cannot rest, but
‘ upon such a certain Point For Example:



‘ Let the Line AB repre-
‘ sent a Plane; and C the
‘ Centre of the Earth, and
‘ that CA be drawn Perpen-
‘ dicular to the Line AB;
‘ I say, That a Bowl being
‘ plac’d upon the Point B, cannot rest there.
‘ For a heavy Body cannot rest, when it may
‘ descend. Now the Bowl B moving towards

‘ A

' A, continually descends, and approaches near
' the Centre of the Earth C; because in the
' Triangle CAB, the Perpendicular CA is shorter
' than BC.

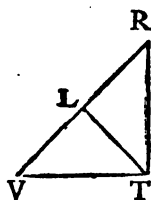
' In like manner we prove, that a Liquid Body
' must flow from B to A, and that its Superficies
' must be round.

PROPOSITION XX.

A THEOREM.

Any two Sides of a Triangle taken together, are greater than the third,

I Say, that the two Sides TL, LV, are greater than the Side TV. Some Men prove this Proposition by the Definition of a Right Line, which is the shortest that can be drawn from one Point to another: Therefore the Line TV is less than the two Lines TL and LV.



But it may also be demonstrated another way. Continue the Side VL to R, so that the Lines LR and LT be equal; then draw the Line RT.

Demonstration.

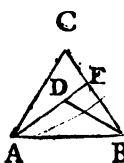
The Sides LT, and LR, of the Triangle LTR, are equal; therefore the Angle R, and LTR,

LTR, are equal ; (*by the 5.*) But the Angle RTV is greater than the Angle RTL : Therefore the Angle RTV is greater than the Angle R : And (*by the 19.*) in the Triangle RTV, the Side RV, that is to say, the Sides LT and LV are greater than the Side TV.

PROPOSITION XXI.

A THEOREM.

If a small Triangle be describ'd within a greater upon the same Base, the Sides of the small one will be less than those of the greater; but they will form a greater Angle.



LET the small Triangle ADB be describ'd within the Triangle ACB, upon the same Base AB. I say, First, The Sides AC and BC are greater than the Sides AD and BD. Continue the Line AD to E.

Demonstration.

In the Triangle ACE, the Sides AC and CE are greater than the Side AE alone, (*by the 20.*) Therefore, adding to them the Side EB; the Sides AC, and CEB, are greater than the Sides AE, and EB. In like manner in the Triangle DBE, the two Sides BE and ED are greater than the Side BD alone; and adding the Side AD, the Sides ADE, and EB, will be greater than AD and BD.

I say

I say further, That the Angle ADB is greater than the Angle ACB: For the Angle ADB is an external Angle, in respect of the Triangle DBE, and therefore greater than the Internal Angle DEB, (*by the 16.*) In like manner the Angle DEB, being an external Angle in respect of the Triangle ACE, is greater than the Angle ACE therefore the Angle ADB is greater than the Angle ACB.

The U S E.

‘ By the help of this Proposition we demonstrate in *Opticks*, that the Base AB viewed
‘ from the Point C, will appear less than when
‘ it is beheld from the Point D; according to
‘ that Principle, That Quantities viewed under a greater Angle, will appear greater.
‘ Therefore ’tis that *Vitruvius* advises, not
‘ much to lessen the Tops of very high Pillars;
‘ because they being so remote from our Sight,
‘ quickly appear slender enough without being
‘ diminished.

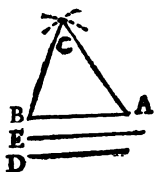
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P R O-

PROPOSITION XXII.

A THEOREM.

To describe a Triangle, whose Sides shall be equal to three Sides given, provided that any two of them be greater than the third.



LET it be propos'd to describe a Triangle, whose Sides shall be equal to three Lines given, AB, D, and E. Measure with the Compass the Line D, and setting one Foot thereof upon the Point B, make an Arch. Then take the Line E, and placing the Foot of your Compass upon the Point A, make another Arch, cutting the former at the Point C: Which done, draw the Lines AC, and BC. I say, That the Triangle ABC is such a one as you desire.

Demonstration.

The Side AC is equal to the Line E, because it reaches to the Arch which is drawn from the Centre A at the Distance of the Line E; and for the same Reason the Side BC is equal to the Line D: Therefore the three Sides AC, BC, and AB, are equal to the Lines E, D, and AB.

I added a Proviso, That the two Lines should be greater than the third; because otherwise, if the Lines D and E were less than the Line AB, the Arches could not cut each other.

The

The USE.

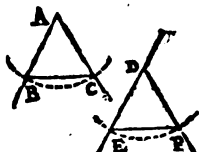
‘ This Proposition may be useful for describing a Figure equal or like to another: For having divided that, which is propos’d to be equall’d, or imitated, into Triangles; and made other Triangles, having equal Sides with the former, we shall have a Figure exactly equal. But if we desire only one that is like, but less, as when we would describe a Plain, or Country, upon Paper: Having divided it into Triangles, and measur’d all their Sides, we must make similar Triangles; giving to each of their Sides so many Parts of a Scale, or Line divided into equal Parts, as the Sides of the Triangles propos’d have of Yards or Feet.

PROPOSITION XXIII.

A PROBLEM.

To make an Angle equal to another at a Point of a Line given.

Suppose you were to make an Angle at the Point A of the Line AB, equal to the Angle EDF. Describe from the Points A and D as Centres, two Arches BC, and EF, at the same Wideness of the Compass; then take the Distance EF, and



and having measured as much in B C, draw the Line A C. I say, the Angles B A C, and E D F, are equal.

Demonstration.

The Triangles A B C, and D E F, have the Sides A B, and A C, equal to the Sides D E, and D F; since the Arches B C and E F were described with the same Wideness of the Compass, the Bases also B C and E F are equal; therefore the Angles B A C and E D F are equal, (by the 8.)

The U S E.

This Problem is so necessary in *Gaodesia*, *Fortifications*, *Perspective*, *Dialling*, and all other Parts of the Mathematicks, that the greatest Part of their Operations would be impossible, if we did not know how to make one Angle equal to another, or of such a Number of Degrees as we please.

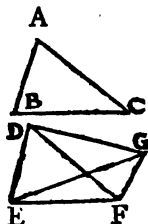
P R O-

PROPOSITION XXIV.

A THEOREM.

Of two Triangles, having each two Sides equal to two of the other; that which has the greatest Angle, has also the greatest Base.

LET the Triangles, ABC, DEF have the Sides AB and DE, AC and DF equal; and let the Angle BAC be greater than the Angle EDF. I say, The Base BC is greater than the Base EF.



Make the Angle EDG equal to the Angle BAC, (*by the 23.*) and the Line DG equal to AC; then draw the Line EG. First, The Triangles ABC and DEG, having the Sides AB and DE, AC and DG, equal, and the Angle EDG equal to the Angle BAC; their Bases BC and EG will be equal, (*by the 4.*) and the Lines DG and DF being both equal to AC, will be equal betwixt themselves.

Demonstration.

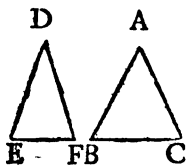
In the Triangle DGF, the Sides DG and DF being equal, the Angles DGF, and DFG will be equal, (*by the 5.*) But the Angle EGF is less than the Angle DGE, and the Angle EFG is greater than the Angle DFG. Therefore in the Triangle EFG, the Angle EFG will be greater

than the Angle EGF: And therefore (*by the 18.*) the Line EG oppos'd to the greater Angle EFG, will be greater than EF. Therefore BC, being equal to EG, is greater than EF.

PROPOSITION XXV.

A THEOREM.

Of two Triangles. having each two Sides equal to two of the other; that which has the greatest Base, has likewise the greatest Angle.



LET the two Triangles ABC, DEF, have the Sides AB, DE, and AC, DF, equal; and let the Base BC be greater than the Base EF. I say, That the Angle A will be greater than the Angle D.

Demonstration.

If the Angle A be not greater than the Angle D; it will be either equal, and then the Bases BC, EF, will be equal, (*by the 4.*) or it will be less, and the Base EF greater than the Base BC, (*by the 24.*) but both are contrary to the Supposition.

∴ These Propositions are necessary to demonstrate those that come after.

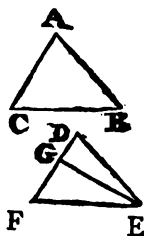
P R O

PROPOSITION XXVI.

A THEOREM.

If one Triangle has one Side, and two Angles, equal to those of another Triangle; 'tis equal to it in all Respects.

LET the Angles ABC, DEF; ACB, DFE, of the Triangles ABC, DEF, be equal; and the Sides BC, and EF, which are between those Angles, also equal. I say, That the other Sides are equal. For Example, AC, and DF; Imagine, if you please, the Side DF to be greater than AC; and cutting GF equal to AC, draw the Line GE.



Demonstration.

The Triangles ABC, GEF, have the Sides EF, BC, AC, GF, equal; the Angle C is also suppos'd to be equal to F. Therefore (*by the 4.*) the Triangles ABC, GEF, are equal in all Respects; and the Angles GEF, and ABC, are equal. But we suppos'd the Angles ABC, DEF, to be equal: And so the Angles DEF, GEF, would be equal; that is, The whole to the Part, which is impossible. Therefore the Side DE will not be greater than the Side AB, nor AC greater than DF, because the same Demonstration may be made in the Triangle ABC.

D 4

Again,

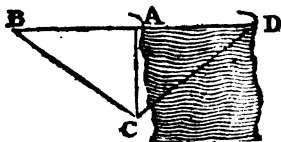
Again, Suppose the Angles A and D, C and F to be equal; and also the Sides BC, and EF, oppos'd to the Angles A and D, to be equal. I say, The other Sides are equal. For if DF be greater than AC, cut GF equal to AC, and draw the Line GE.

Demonstration.

The Triangles ABC, GEF, having the Sides EF, BC, FG, CA, equal, will, (*by the 4.*) be equal in all Respects: And the Angles EGF, BAC, will be equal. But we suppos'd, that the Angles A and D were equal, therefore the Angles D, and EGF, must be equal, which is impossible, since the Angle EGF, being the external Angle in respect of the Triangle EGD, must be greater than the internal D, (*by the 16.*) therefore the Side DF is not greater than AC.

The USE.

‘ *Thales* made use of this Proposition, to measure inaccessible Distances. For Example:



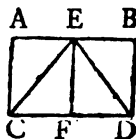
‘ The Distance AD
‘ being propos'd, he
‘ would draw from the
‘ Point A, the Line
‘ AC Perpendicular to
‘ AD: Then describing a Semi-circle at the
‘ Point C, would measure the Angle ACD,
‘ and take another equal to it on the other Side,
‘ prolonging the Line CB 'till it meet with the
‘ Line DA at the Point B; and then demonstrate
‘ the

the Lines AD and AB to be equal; so that measuring the Line AB, which was accessible, he could know the other which was not. For the two Triangles ADC, and ABC, have the right Angles CAD and CAB equal, the Angles ACD, and ACB are also taken equal; and the Side AC is common to both: Therefore (*by the 26.*) the Sides AD and AB are equal.

A L E M M A.

A Line which is Perpendicular to one of two Parallels, is also Perpendicular to the other.

LET the Parallel Lines be AB and CD, and let EF be Perpendicular to CD. I say, 'Tis also Perpendicular to AB. Cut the Line CF equal to FD, and upon the Points C and D raise two Perpendiculars to CD, which, *by the Definition of Parallels*, will be equal to FE; then draw the Lines EC and ED.



Demonstration.

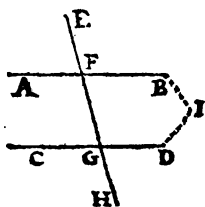
The Triangles CEF, and DEF, have the Side FE common, the Sides CF, FD, equal, and the Angles CFE, and DFE, Right, and by consequence equal; therefore (*by the 4.*) the Bases EC, ED, and the Angles FED, FEC, and FCE, FDE, will be equal; the two last of which being taken away from the right Angle ACF and BDF, leave the two Angles ACE,

‘ ACE, and BDE, equal: Therefore the Tri-
 ‘ angles CAB, DBE, will have (*by the 4.*) the
 ‘ Angles DEB, CEA, equal; which being ad-
 ‘ ded to the equal Angles CEF, DEF, make the
 ‘ Angles FEB, and FEA, equal; therefore the
 ‘ Line EF is Perpendicular to AB.

PROPOSITION XXVII.

A THEOREM.

If a Line, falling upon two others, makes with them the Alternate Angles equal, those two Lines are parallel.



LET the Line EH, falling upon the Lines AB and CD, make with them the alternate Angles AFG, FGD equal. I say, First, The Lines AB, and CD, will never concur, tho' continu'd as far as you please: For suppose them to concur in I, and that FBI, and CDI are two right Lines.

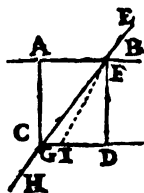
Demonstration.

If FBI, and GDI, be two right Lines, FIG is a Triangle; and (*by the 16.*) the external Angle AFG, is greater than the Internal FGI. They cannot therefore be equal, if the Lines AB and CD ever concur.

‘ But

‘ But because we have Examples of some
‘ crooked Lines, which never concur; and yet
‘ are not Parallels, approaching still nearer and
‘ nearer to each other.

I say, Secondly, That if the the Line EH, fall-
ing upon the Lines AB, and CD,
makes the alternate Angles AFG,
and FGD, equal; the Lines AB,
CD, are Parallel, or in all Re-
spects equally remote from each
other, so that the Perpendiculars
between them will be equal. From
the Point G draw the Perpendicu-
lar GA, to the Line AB; and taking GD equal
to AF, draw FD.



Demonstration.

The Triangles AGF, and DFG, have the Side
FG common; the Side GD is also taken equal
to the Side AF, and the Angles AFG and FGD
suppos'd to be equal: Therefore (*by the 4.*) the
Bases AG and FD are equal, and the Angle
CDF is equal to the right Angle CAB; there-
fore FD is Perpendicular. I add, That the
Line AB is Parallel to CD: For the only Parallel
Line that can be drawn from the Point F to the
Line CD, ought to pass by the Point A, ac-
cording to the Definition of Parallels; which
requires, that the Perpendicular Lines AG and
FD be equal.

PROPOSITION. XXVIII.

A THEOREM.

If a Line, falling upon two others, makes the external Angle equal to the internal opposite Angle on the same Side; or the two internal Angles on the same Side equal to two Right ones, those two Lines will be Parallel.

IN the precedent Figure, suppose the Line EH, falling upon AB and CD, to make first the external Angle EFB equal to the internal opposite Angle on the same Side FGD: I say, That the Lines AB and CD are Parallel.

Demonstration.

The Angle EFB is equal to the Angle AFG, being oppos'd to it at the Top, (*by the 15.*) and 'tis suppos'd that the Angle FGD is also equal to the Angle EFB: Therefore the alternate Angles AFG, FGD, will be equal; and (*by the 27.*) the Lines AB, and CD, will be Parallel.

I say, In the second Place, That if the Angles BFG, and FGD, which are the internal Angles on the same Side, be equal to two Right ones, the Lines AB and CD will be Parallel.

Demonstration.

The Angles AFG and BFG are equal to two right Angles, (*by the 13.*) and 'tis suppos'd that the Angles BFG, and FGD are also equal to two
right

right Angles; therefore the Angles AFG, BFG, are equal to the Angles BFG, and FGD; therefore taking away the Angle BFG, which is common to both, the alternate Angles AFG and FGD will be equal; and (*by the 27.*) the Lines AB and CD will be Parallel.

PROPOSITION XXIX.

A THEOREM.

If a Line cut two Parallels, the alternate Angles will be equal; the external Angle will be equal to the Internal Opposite Angle; and the two Internals on the same Side will be equal to two right Angles.

LET the Line EH [*see Fig. preced.*] cut the two Parallels AB, and CD; I say, First, The alternate Angles AFG, and FGD, are equal. From the Points F and G draw the Perpendiculars GA, and FD, which by the Definition of Parallels are equal.

Demonstration.

In the Rectangle Triangles AFG, and FGD, the Sides FD and AG being equal, as also the Right Angles A and D, and the Side FG common to both: I say, First, That the Side GD is equal to AF. For if GD be greater; having cut the Line DI equal to AF, and drawn the Line FI; the Triangles AFG and FDI would have

have their Bases GF and FI equal, which is impossible. For since the Angle D is a right Angle, the Angle FID is an Acute, and FIG an Obtuse, (*by the 13.*) Therefore (*by the 18.*) in the Triangle FIG , the Side FG oppos'd to the Obtuse Angle, is greater than FI . Therefore DG is equal to AF ; and the Triangles AFG and FGD , having all their Sides equal, will have the alternate Angles AFG and FGD equal, as being oppos'd to the equal Sides AG , and FD .

I say again, That the external Angle EFB is equal to the external EGD ; because (*by the 15.*) it is equal to its opposite Angle AFG , which is equal to its alternate FGD .

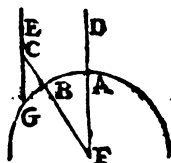
Lastly, Since the Angles AFG and GFB are equal to two Right ones; taking away AFG , and substituting in its place its alternate FGD , the two internal Angles GFB , and FGD , will be equal to two right Angles.

The USE.

‘ *Eratosthenes* found out, by these Propositions,
 ‘ a way of measuring the Circuit or Circum-
 ‘ ference of the Earth. In order to which, he
 ‘ suppos'd two Rays, proceeding from the Centre
 ‘ of the Sun to two Points of the Earth, to be
 ‘ Physically parallel; and also that at *Syene*, a
 ‘ Town in the higher Parts of *Egypt*, the Sun
 ‘ comes exactly to the Zenith upon the Day of
 ‘ the Solstice, observing the Wells there to be
 ‘ then illuminated to the very Bottom; and
 ‘ likewise

likewise computed the Distance between *Alexandria* and *Syene*, by Miles or Furlongs.

Let us therefore suppose *Syene* to be at the Point A, and *Alexandria* at B, where we erect a Style BC Perpendicular to the Horizon; and let the two Lines DF and EG repre-

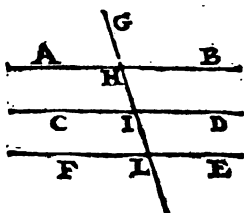


sent two Rays proceeding from the Centre of the Sun upon the Day of the Solstice, which are parallel to each other. DA, which passes by *Syene*, is Perpendicular, that is, it passes through the Centre of the Earth. Having observed by the Perpendicular Style BC the Angle GCB, made by the Ray of the Sun EG; I say, The Rays DA and EG being parallel, the alternate Angles GCB and BFA are equal : By which means we have got the Angle AFB, and its Measure AB; which gives us in Degrees the Distance between *Alexandria* and *Syene*. And having suppos'd it to be known in Miles, the Circumference of the Earth may be found by the simple Rule of Three direct; saying, If so many Degrees give so many Miles, how many will 360 give?

PROPOSITION XXX.

A THEOREM.

Lines parallel to a third, are also parallel among themselves.



Suppose the Lines AB and FE to be parallel to the Line CD: I say, they are parallel betwixt themselves. Let the Line AL cut them all three.

Demonstration.

Forasmuch as the Line AB and CD are parallel, the alternate Angles AHI, and HID, are equal, (*by the 29.*) and because the Lines CD and FE are also parallel, the external Angle HID will be equal to the internal ILE, (*by the same:*) Therefore the alternate Angles AHI, and ILE, will be equal, and the Lines AB and FE parallel, (*by the 27.*)

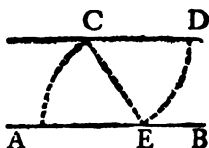
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PROPOSITION XXXI.

A PROBLEM.

To draw a Line parallel to another by a Point given.

LET it be requir'd to draw a Line by the Point C, which shall be parallel to the Line AB. Draw the Line CE, and make the Angle ECD equal to the Angle CEA: I say, the Line CD is Parallel to AB.

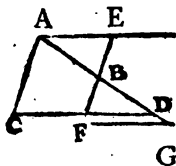


Demonstration.

The alternate Angles DCE and AEC are equal: Therefore the Lines CD and AB are Parallels.

' The eleventh Maxim, i. e. If a Line falling upon two others makes the internal Angles less than two right Angles, those Lines will concur, may also now be easily demonstrated.

Let the Line AC, falling upon the Lines AB and AD, make the internal Angles ACD, and CAB, less than two right Angles: I say, the Line AB and CD will concur. Let the Angles ACD and CAE be equal to two right Angles: The Lines AE and CD will be Parallels, (by the 28.) Take the Line AB as long as you please, and by the Point B draw FE parallel



Parallel to CA. Then take the Line EB so oft as is necessary, to make it reach lower than the Line CD; as in the present Figure I have taken it only twice; so that EB and BF are equal. By the Point F draw a Parallel FG equal to AE, and join the Line GB. I say, That the Line ABG is only one Line; and that therefore the Line AB concurring in FG, if the Line CD be continued, since it cannot cut its Parallel FG, it will cut the Line BG between B and G.

Demonstration.

The Triangles AEB and BFG have the Sides AE and FG, BE and BF, equal; as also the alternate Angles AEB, and BFG, (*by the 29.*) therefore they are equal in all Respects, (*by the 4.*) And the opposite Angles ABE, FBG, are equal; and by consequence (*by the Coroll. of the 15.*) AB and BG make but one right Line.

The USE.

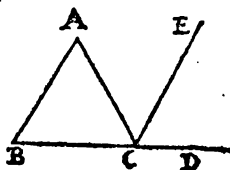
‘ The Use of Parallel Lines is very common;
 ‘ as in *Perspectives*, forasmuch as the Appearances
 ‘ or Images of Lines parallel to the Picture or
 ‘ Table, are parallel among themselves. In
 ‘ *Navigation*, the Lines of the same Rhomb of
 ‘ the Wind are described by Parallels. *Polar*
 ‘ *Dials* have the Hour-lines Parallels. The
 ‘ Compass of Proportion is founded also upon
 ‘ Parallels.

PROPOSITION XXXII.

A THEOREM.

The external Angle of a Triangle is equal to both the internal opposite Angles, taken together; and all the three Angles of a Triangle are equal to two right Angles.

LET the Side BC of the Triangle ABC be produc'd to D. I say, That the external Angle ACD is equal to both the internal Angles A and B taken together. By the Point C draw the Line CE parallel to the Line AB.



Demonstration.

The Lines AB and CE are Parallels; therefore, (*by the 29.*) the alternate Angles ECA and BAC are equal; and (*by the same*) the external Angle ECD is equal to the internal B. And by consequence the whole Angle ACD, being equal to both the Angles ACE, and ECD, of which it is compos'd, will be equal to both the Angles A and B taken together.

In the second place: The Angles ACD and ACB are equal to two right Angles, (*by the 13.*) and I have demonstrated the Angle ACD to be equal to both the Angles A and B taken together;

E 2

therefore

64 *The Elements of Euclid.*

therefore the Angles ACB, A, and B, that is to say, All the Angles of the Triangle ABC, are equal to two right Angles, or, which is all one, to 180 Degrees.

Corollary 1. All the three Angles of one Triangle are equal to all the three Angles of another Triangle.

Coroll. 2. If two Angles of one Triangle be equal to two Angles of another Triangle, their third Angles are also equal.

Coroll. 3. If a Triangle has one right Angle, the other two will be Acute; and taken together, will be equal to one right Angle.

Coroll. 4. From a Point given, only one Perpendicular can be drawn to the same Line; because a Triangle cannot have two right Angles.

Coroll. 5. A Perpendicular is the shortest of all the Lines that can be drawn from the same Point to the same Line.

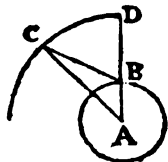
Coroll. 6. In a Rectangle Triangle the right Angle is the greatest Angle; and the Side oppos'd to it the greatest Side.

Coroll. 7. Every Angle of an equilateral Triangle contains 60 Degrees; That is to say, The third Part of 180.

The USE.

‘ This Proposition is of Use in *Astronomy*, to determine the *Parallax*. Suppose the Point A to be the Centre of the Earth; and that
‘ from

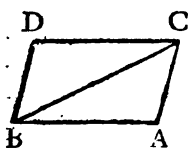
' from the Point A upon the
 ' Superficies be taken the Angle
 ' DBC , that is to say, the Di-
 ' stance of a Star from the *Zenith*
 ' D. If the Earth was transpa-
 ' rent, the Star would appear re-
 ' mote from the *Zenith* D, according to the
 ' Bigness of the Angle CAD , which is less than
 ' the Angle CBD . For the Angle CBD being
 ' the external Angle in respect of the Triangle
 ' ABC , it is (*by the 32.*) equal to both the op-
 ' posite Angles A and C. Therefore the Angle
 ' C will be equal to the Excess of the Angle
 ' CBD above the Angle A. Whence I infer,
 ' That if I can know by the *Astronomical Tables*
 ' how far remote from the *Zenith* the Star ought
 ' to appear to him that should be at the Centre
 ' of the Earth, and observe it at the same time
 ' from the *Superficies*, the Difference of those
 ' two Angles will be the *Parallax* BCA .



PROPOSITION XXXIII.

A THEOREM.

Two Lines drawn towards the same Parts, from the Extremities of two other Lines that are equal and parallel, are also themselves equal and parallel.



LET the Line AB and CD be parallel and equal; and let the Lines AC and BD be drawn from their Extremities towards the same Parts: I say, That the Lines AC and BD are equal and parallel. Draw the Diagonal Line BC.

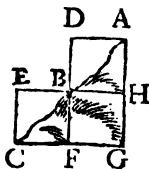
Demonstration.

Since the Lines AB and CD are parallel, the alternate Angles ABC and BCD will be equal (*by the 29.*) Therefore in the Triangles ABC and BCD, which have the Side BC common, and the Sides AB and CD equal, together with the Angles ABC and BCD equal also, the Bases AC and BD will be equal, (*by the 4.*) and also the Angles DBC, and BCA; which being alternate Angles, the Lines AC and BD will be parallel, (*by the 27.*)

The USE.

‘ This Proposition is reduc’d to Practice for
 ‘ the measuring the perpendicular Heights,
 ‘ AG,

AG, of the vastest Mountains;
and also their horizontal Lines,
CG, which are hid by their Bulk.
Take a large Square ADB, and
place it so at the Point A, that
the Side DB may fall perpendi-
cularly; then measure the Sides AD and DB.
This done, do the same again at the Point B,
and measure BE and EC: The Sides parallel
to the Horizon AD, BE, added together, give
the Horizontal Line CG; and the perpendi-
cular Sides DB and EC, give the perpendi-
cular Height AG. This way of Measuring
is call'd * *Cultellation*.



* *Measuring by Piecemeal.*

PROPOSITION XXXIV.

A THEOREM.

The opposite Sides and Angles of a Parallelogram are equal; and the Diameter divides it into two equal Parts.

Suppose the Figure ABDC [See the Fig. of the preceding Prop.] to be a Parallelogram, that is to say, that the Sides AB, CD; AC, and BD, are parallel. I say, The opposite Sides AB, CD; AC and BD, are equal; as also that the Angles A and D, ABD and ACD; and that the Diameter BC equally divides the whole Figure.

Demonstration.

The Lines AB, and CD, are suppos'd to be Parallels: Therefore the alternate Angles ABC and BCD will be equal, (*by the 29.*) In like manner the Sides AC and BD being suppos'd to be Parallels, the alternate Angles ACB and DCB will be equal. And further, The Triangles ABC, and DCB, having the same Side BC; and the Angles ABC, BCD; ACB, and CBD equal, will be equal in all respects, (*by the 28.*) Therefore the Sides AB, CD; AC and BD, and the Angles A and D, are equal: And the Diameter divides the Figure into two equal Parts. And since the Angles ABC, BCD, ACB and CBD, are equal, joining together ABC and CBD; and likewise BCD and ACB, we infer that the opposite Angles ABD and ACD are equal.

The USE.

‘ Surveyors have need of this
 ‘ Proposition for dividing Grounds.
 ‘ If the Field be a Parallelogram,
 ‘ they can divide it into two equal
 ‘ Parts by the Diameter AD. But
 ‘ if you be oblig’d to divide it by the Point E;
 ‘ divide first the Diameter into two equal Parts
 ‘ by the Point F, then draw the Line EFG,
 ‘ which will divide the Figure into two equal
 ‘ Parts. For the Triangles AEF and GFD,
 ‘ having the alternate Angles EAF, GDF;
 ‘ and

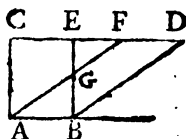
and AEF, FGD; and the Sides AF and FD equal, are equal (*by the 26.*) and since the Trapezium BEFD with the Triangle AEF; that is to say, The Triangle ADB, is half the Parallelogram, (*by the 34.*) the same Trapezium BEFD with the Triangle DGF will be half the same. Therefore the Line EG divides it in the Middle.

PROPOSITION XXXV.

A THEOREM.

Parallelograms, having the same Base, and being between the same Parallels, are equal.

LET the Parallelograms be ABEC, and ABDF, having the same Base AB, and being between the same Parallels AB and CD: I say, They are equal.



Demonstration.

The Sides AB, CE, are equal (*by the 34.*) as also AB, FD: Therefore CE and FD are equal; and adding to them EF, the Lines CF and ED will be equal. The Triangles therefore CFA, and EDB, have the Sides CA, EB, as also CF, and ED, equal together with the Angles DEB, and FCA, (*by the 29.*) one being an external, and the other an internal Angle, on the same

Same Side: Therefore (by the 4.) the Triangle ACF and BED are equal; and taking from them both, that which is common, viz. the little Triangle EGF, the Trapezium FGBD will be equal to the Trapezium CAGE; and adding to both the Triangle AGB, the Parallelograms ABEC and ABDF will be equal.

The U S E.

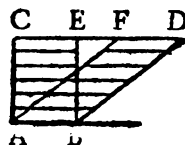
‘ Scotus, and some Divines since him, have
 ‘ made use of this Proposition, to prove, that
 ‘ Angles may extend themselves to what Space they
 ‘ please. For, supposing they can assume any
 ‘ Figure, provided they have not a greater Ex-
 ‘ tension; it is evident, That if an Angle
 ‘ should possess the Space of a Parallelogram
 ‘ ABEC, it may likewise occupy the Space of
 ‘ the Parallelogram ABDF; and because Pa-
 ‘ rallels may be continued *in infinitum*, (with-
 ‘ out end,) and Parallelograms may be still
 ‘ form’d longer and longer, which will all be
 ‘ equal to ABEC; an Angle will be able to ex-
 ‘ tend itself still farther and farther.

*A Demonstration of the same Proposition by the
 Method of Indivisibles.*

‘ This Method was lately invented by Cava-
 ‘ lerius; which has found different Acceptation
 ‘ in the World, some approving, and others re-
 ‘ jecting it. His Method consists in this;
 ‘ That we imagine Superficies to be compos’d
 ‘ of

of Lines like so many Threads. And 'tis certain, that two Pieces of Linnen will be equal, if they have both the same Number of Threads, of equal length, and equally compacted.

Let two Parallelograms therefore ABEC, and ABDF be propos'd, having the same Base AB, and being between the same Parallels AB, CD: Divide the Parallelogram



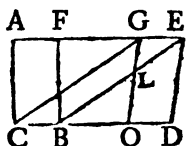
ABEC into as many Lines as you please, parallel to AB, which continue to the other Parallelogram ABDF. 'Tis evident, There will be no more in one, than in the other; and that they will be of equal Length, being all equal to the Base AB; and that they will not be more closely compacted in one, than in the other; therefore the Parallelograms will be equal.

P R O-

PROPOSITION. XXXVI.

A THEOREM.

Parallelograms, upon equal Bases, and between the same Parallels, are equal.



LET the Bases CB and OD of the Parallelograms ACBF, ODEG, be equal; and let both be between the same Parallels AE, CD. I say, The Parallelograms are equal, draw the Lines CG, and BE.

Demonstration.

The Bases CB, and OD, are equal: OD, and GE, are also equal: Therefore CB and GE are equal, and parallel; and by consequence (*according to the 33.*) CG and BE will be equal and parallel; and CBEG will be a Parallelogram equal to CBFA, (*by the 35.*) having both the same Base. In like mannner, taking EG for the Base, the Parallelograms GODE and CBEG will be equal, (*by the same.*) Therefore the Parallelograms ACBF and ODEG are equal.

The USE.

‘ We oft reduce Parallelograms, which have
 ‘ Oblique Angles, as CBEG, or ODEG, to
 ‘ Rectangles; as CBFA: So that measuring the
 ‘ latter, which is easie, being only to multiply
 ‘ AC

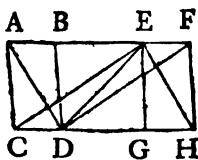
‘ AC by CB, the Product being equal to the
‘ Parallelogram ACBF, we may by conse-
‘ quence know the other Parallelograms CBEG,
‘ or ODEG.

PROPOSITION XXXVII.

A THEOREM.

Triangles having the same Base, and being between the same Parallels, are equal.

IF the Triangle ACD and CDE, have the same Base CD, and be inclos'd between the same Parallels AF, and CH, they will be equal. Draw the Lines DB, and DF, parallel to the Lines AC, and CE, and you will have form'd two Parallelograms.



Demonstration.

The Parallelogram ACDB, and ECDF, are equal, (*by the 35.*) and the Triangles ACD, CDE, are the Halfs of those Parallelograms, (*by the 34.*) Therefore the Triangles ACD, CDE, are equal.

PROPOSITION XXXVIII.

A THEOREM.

Triangles, that have equal Bases, and are inclos'd within the same Parallels, are equal.

IF the Triangles ACD and EGH, [see Fig. *preced.*] have equal Bases CD, and GH, and are inclos'd within the same Parallels AF and CH, they are equal. Draw the Lines BD and HF parallel to the Sides AC, and EG; and you will have form'd two Parallelograms.

Demonstration.

The Parallelograms ACDB, and EGHF, are equal, (*by the 36.*) and the Triangles ACD and EGH are the Halfs of those Parallelograms, (*by the 34.*) therefore they are also equal.

The USE.

We have in this Proposition Directions for dividing a Triangular Field into two equal Parts; for Example, the Triangle ABC. Divide the Line which you will take for the Base, as BC, into two equal Parts in D: I say, The Triangles ABD, and ADC, are equal. For if you suppose a Line drawn by A, parallel to BC, those Triangles will have equal Bases, and be inclos'd within the same Parallels: and by consequence will be



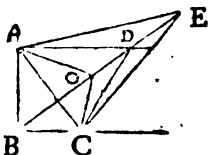
be equal. Other Divisions, grounded upon the same Proposition, might be made; but I omit them, that I might not be tedious.

PROPOSITION XXXIX.

A THEOREM.

Equal Triangles upon the same Base, are within the same Parallels.

IF the Triangles ABC , and DBC , having the same Base BC , be equal; the Line AD drawn by the Tops will be parallel to the Base BC . For if AD and BC be not parallel; if you draw a Parallel by the Point A , it will fall either below the Line AD , as AO ; or above it, as AE . Suppose it to fall above, and produce BD 'till it meet the Line AE at the Point E ; then draw the Line CE .



Demonstration.

The Triangles ABC and EBC are equal, (by the 37.) since the Lines AE and BC are Parallel; 'tis likewise suppos'd that the Triangles ABC , and DBC are equal: Therefore the Triangles DBC and EBC would be equal: which is impossible, the first being part of the second. Whence I conclude, that a Line parallel to BC cannot be drawn above AD , as AE .

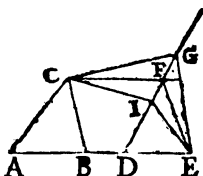
I add,

I add, That a Parallel cannot be below AD, as AO: Because the Triangle BOC would be equal to the Triangle ABC, and by consequence to the Triangle DBC; that is to say, The Part would be equal to the Whole. It must therefore be confes'd, that the Line AD is parallel to the Line BC.

PROPOSITION XL.

A THEOREM.

Equal Triangles, having equal Bases, if they be taken upon the same Line, are between the same Parallels.



IF the equal Triangles ABC and DEF, have equal Bases AB and DE; taken upon the same Line AE; the Line CF drawn by their Tops, will be parallel to AE. For, if it be not parallel, having drawn by the Point C a Line parallel to AE, it will pass either above CF, as CG; or below it, as CI.

Demonstration.

If it pass above CF, as CG, continue DF till it meet with CG in G; and draw the Line EG. The Triangles ABC and DEG would be equal, (*by the 38.*) and ABC and DEF being suppos'd to be equal, DEF, DEG, would be also equal;

equal; which, one being part of the other, cannot possibly be: Therefore the Parallel cannot pass above CF.

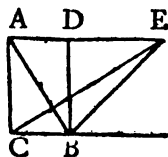
I add, that neither can it pass below it, as CI; because then the Triangles ABC and DEI would be equal; and by consequence DEI, and DEF; the Part and the Whole: Therefore only CF can be parallel to AE.

PROPOSITION XLI.

A THEOREM.

A Parallelogram will be double to a Triangle, if they be between the same Parallels, and have equal Bases.

IF the Parallelogram ACBD, and the Triangle EBC, be between the same Parallels AE and BC; and have the same Base BC, or only equal Bases; the Parallelogram will be double the Triangle. Draw the Line AB.



Demonstration.

The Triangles ABC and BCE are equal, (by the 37.) But the Parallelogram ACBD is double the Triangle ABC, (by the 34.) It is therefore double the Triangle BCE. It would be also double a Triangle, that, having a Base equal to BC, should be between the same Parallels.

F

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The USE.

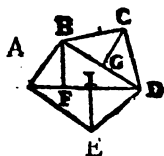
H A G



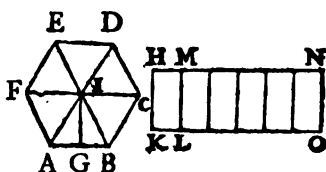
‘ The ordinary Method of Measuring the *Area* or Superficies of a Triangle is built upon this Proposition. If the Triangle ABC be propos’d; from the Angle A we must draw AD perpendicular to the Base BC; then multiplying the Perpendicular AD by half the Base BE, the Product gives the *Area* of the Triangle; because multiplying AD, or, what is the same, EF by BE we have a Rectangle BEFH, which is equal to the Triangle ABC. For (*by the 41.*) the Triangle ABC is half the Rectangle HBCG; and so likewise is the Rectangle BEFH.

‘ We measure all sorts of Rectilineal Figures, as ABCDE, by dividing them into Triangles, as BCD, ABD, AED; drawing the Lines AD, and BD; and the Perpendiculars CG, BF, and EI. For multiplying half of BD by CG, and half of AD by BF, and by EI, we have the *Area* of all those Triangles: Adding which together, the Sum is equal to the Rectilineal Figure ABCDE.

‘ We find that *Area* of regular Polygons, by multiplying half their Circuit by a Perpendicular



perpendicular drawn
from their Centre
to one of their
Sides. For multi-
plying AG by IG,
we shall have a



Rectangle HILM equal to the Triangle AIB:
And repeating the same for all the other Tri-
angles, taking always half of the Bases, we
shall have a Rectangle HKON, which will
have the Side KO compounded of all the half
Bases, and by consequence equal to half the
Circumference, and the Side HK equal to the
Perpendicular IG.

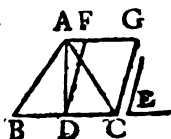
'Tis according to this Principle, that *Archimedes* has demonstrated, that a Circle is equal to a Rectangle compris'd under the Semi-diameter, and a Line equal to half the Circumference.

PROPOSITION XLII.

A PROBLEM.

To make a Parallelogram equal to a Triangle given, having one Angle equal to an Angle given.

LET a Parallelogram be desired equal to the Triangle ABC, and having an Angle equal to the Angle E. Divide the Base BC into two equal



F 2

Parts

Parts at the Point D; and draw the Line AG parallel to BC (*by the 31.*) then, make the Angle CDF equal to E, (*by the 23.*) And lastly, Draw the Line CG parallel to DF: The Figure FDCG is a Parallelogram, because the Lines FG, DC; FD and GC, are Parallels; and its Angle CDF is equal to the Angle E; and further, 'tis also equal to the Triangle ABC.

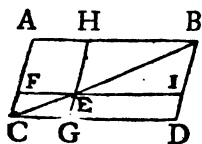
Demonstration.

The Triangle ADC is half the Parallelogram FDCG, (*by the 41.*) 'tis also half the Triangle ABC; since the Triangles ADC and ADB are equal (*by the 38.*) Therefore the Triangle ABC is equal to the Parallelogram FDCG.

PROPOSITION XLIII.

A THEOREM.

The Complements of a Parallelogram are equal.



IN the Parallelogram ABDC, the Complements AFEH, and EGDI, are equal.

Demonstration.

The Triangles ABC, and BCD, are equal, (*by the 34.*) Therefore if the Triangles HBE, and BIE; FEC, and CGE, which are also equal (*by the same*) be subtracted, the Complements AFEH, GDIE, which remain, will be equal.

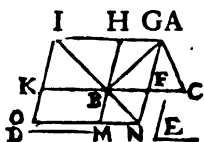
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PROPOSITION XLIV.

A PROBLEM.

To describe a Parallelogram upon a Line given, which shall be equal to a Triangle given, and have such a certain Angle; i. e. equal to one given.

SUPPOSE you be requir'd to make a Parallelogram which shall have one of its Angles equal to the Angle E, and one of its Sides equal to the Line D, and be equal to the Triangle ABC. Make the Parallelogram BFGH, (*by the 42.*) which has the Angle BFG equal to the Angle E, and is equal to the Triangle ABC. Produce the Sides GF, and GH, so that HI may be equal to the Line D; and draw the Line IBN 'till it cuts GF, produc'd to N; and from the Point N draw the Line NO parallel to GI, and IO parallel to BH; producing also the Side FB to K, and HB to M: The Parallelogram MK is that which you desire.

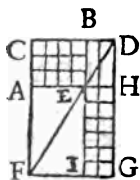


Demonstration.

GF and HM being Parallels, the alternate Angles GFB or the Angle E, and FBM, are equal (*by the 29.*) In like manner the Lines KB and MN being parallel, the alternate Angles FBM, and BMO, are equal; therefore the Angle BMO is equal to E, and the Side KB is equal

to the Line HI, or D: And lastly, The Parallelogram MK is equal to the Parallelogram GFBH, (*by the preced.*) and that was made equal to the Triangle ABC: Therefore the Parallelogram MK is equal to the Triangle ABC.

The USE.



‘ This Proposition contains a
 ‘ kind of *Geometrical Division*:
 ‘ For in *Arithmetical Division* a
 ‘ Number is propos’d, which may
 ‘ be looked on as a Rectangle: For
 ‘ Example, the Rectangle AB, con-
 ‘ sisting of twelve Square Feet, which is to be
 ‘ divided by another Number, suppose two;
 ‘ that is to say, another Rectangle is desir’d to
 ‘ be made equal to AB, having one of its Sides,
 ‘ BD, equal to two; and the Question is, how
 ‘ many Feet the other Side ought to contain;
 ‘ which is, as it were, the Quotient. This is
 ‘ done Geometrically by the Rule and Com-
 ‘ pass, thus; Take BD consisting of two Feet,
 ‘ and draw the Diagonal DEF: The Line AF is
 ‘ that which you seek. For, having compleat-
 ‘ ed the Rectangle DCFG, the Complements
 ‘ EG, and EC, are equal, (*by the 43.*) and EG
 ‘ has for one of its Sides EH, equal to BD, of
 ‘ two Feet in lenth; and EI equal to AF. This
 ‘ kind of Division is call’d *Application*, because
 ‘ the Rectangle AB is apply’d to the Line BD
 ‘ or EH: And from hence ’tis, that Division is
 ‘ frequently call’d *Application*; because the an-
 ‘ cient

‘cient Geometricians made more Use of the
‘ Rule and Compass, than of *Arithmetick*.

PROPOSITION XLV.

A PROBLEM.

To describe a Parallelogram, which shall have a certain Angle; and be equal to a Rectilineal Figure given.

LET the Rectilineal Figure propos'd be $ABCD$, to which you are required to make an equal Parallelogram, which shall have an Angle equal to the Angle E . Divide the Rectilineal into Triangles by the Line BD : And (by the 42.) make a Parallelogram $FGHI$, which has the Angle IHG equal to the Angle E , and is equal to the Triangle ABD ; and (by the 44.) make the Parallelogram $IHKL$ equal to the Triangle BCD , having one Side equal to IH , and the Angle LIH equal to the Angle E . The Parallelogram $FGKL$ will be equal to the Rectilineal $ABCD$.



Demonstration.

Nothing need be prov'd, but that the Parallelograms $FGHI$, and $IHLK$, make up but one; that is to say, GH and HK , make but one right Line. The Angles GHI , and LKH , are equal

to the Angle E. And the Angles LKH, and KHI, are equal to two right Angles, because KHIL is a Parallelogram: Therefore the Angles GHI and KHI are equal to two right Angles, and (*by the 14.*) GH and HK make one right Line.

The USE.

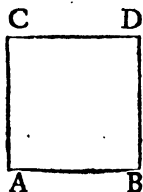
‘ The Use of this Proposition is the samewith
 ‘ the preceding; both serving to measure the
 ‘ Capacity of any Figure whatsoever, by reducing it into Triangles, and then making a
 ‘ Rectangle Parallelogram equal to them.

‘ ’Tis easie likewise to make a Rectangular
 ‘ Parallelogram upon a determinate Side, which
 ‘ may be equal to many irregular Figures. In
 ‘ like manner, having many Figures, a Rectangle may be describ’d equal to their Difference.

PROPOSITION XLVI.

A PROBLEM.

To describe a Square upon a Line given.



TO describe a Square upon the Line AB, draw two Perpendiculars AC and BD equal to AB, and draw the Line CD.

Demonstration.

The Angles A and B being right Angles,

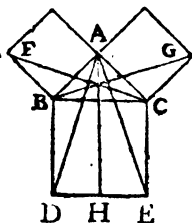
Angles, the Lines AC and BD are Parallels, (*by the 28.*) They are also equal; therefore the Lines AB and CD are Parallels, and equal, (*by the 33.*) and the Angles A and C equal to two right Angles; as also B and D, (*by the 29.*) and since A and B are both right Angles, the Angles C and D will be so likewise: Therefore the Figure AD has all its Sides equal, and all its Angles, right Angles, and by consequence is a Square.

PROPOSITION XLVII.

A THEOREM.

The Square of the Base of a Rectangle Triangle, is equal to the Squares of both the other Sides taken together.

SUPPOSE the Angle ABC to be a right Angle, and that Squares were described upon all the Sides BC, AB, and AC: That upon the Base BC, which is oppos'd to the right Angle, will be equal to the Squares of both the Sides AB, and AC.



Draw the Line AH parallel to BD and ~~GE~~; and join the Lines AD, AE, FC, and BG. I will prove the Square AF is equal to the Rectangle BH, and the Square AG to the Rectangle CH; and therefore the Square BE is equal to both the Squares AF and AG.

Demon-

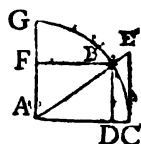
Demonstration.

The Triangles FBC, and ABD, have the Sides AB, BF; BD and BC equal: And the Angles FBC, and ABD, are equal, each containing the Angles ABC more than their respective right Angles. Therefore (*by the 4.*) the Triangles ABD and FBC are equal. But the Square F is double the Triangle FBC, (*by the 41.*) having the same Base BF, and being between the same Parallels BF, and AC. In like manner the Rectangle BH is double the Triangle ABD, having likewise the same Base BD, and being between the same Parallels BD and AH. Therefore the Square BF is equal to the Rectangle BH. By the same Method the Triangles ACE, and GCB may be prov'd to be equal, (*by the 4.*) and the Square AG to be double the Triangle GCB; and the Rectangle CH, double the Triangle ACE, (*by the 41.*) therefore the Square AG is equal to the Rectangle CH; and by consequence the Squares AF and AG are equal to the Square BDEC.

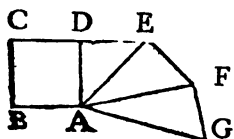
The U S E.

' 'Tis said, that *Pythagoras*, having found out
 ' this Proposition, sacrific'd a *Hecatomb*, i. e.
 ' a hundred Oxen, to the *Muses*, to return them
 ' Thanks for their Assistance; supposing it,
 ' it seems, above the Power of bare Human In-
 ' vention. Nor was his Esteem thereof so
 ' irrational, as to some perhaps it may appear;
 ' this

this Proposition being the Foundation of a
 very considerable part of the *Mathematicks*.
 For in the first place, *Trigonometry* cannot
 possibly subsist without it, it being necessary
 to compose a Table of all the Lines that may
 be inscrib'd in a Circle, as Chords, Sines,
 Tangents, Secants; as will appear by one
 Example: Suppose the Semi-
 diameter AC to be divided into
 100000 Parts, and that the Arch
 BC contains 30 Degrees. Since
 the Chord, or Line that subtends
 60 Degrees is equal to the Semi-diameter AC;
 BD the Sine of 30 Degrees, will be equal to
 half AC, and therefore contain 50000 Parts.
 Now in the Rectangular Triangle ADB, the
 Square of AB is equal to the Squares of AD
 and BD. Make therefore the Square of AD,
 by multiplying 100000 by 100000, and from
 the Product subtract the Square of 50000 or
 BD; the Remainder will be the Square of
 AD, or FB the Sine of the Complement;
 and extracting the Square Root of that Num-
 ber, you will have the Line FB. This done,
 making as AD to BD, so AC to CE, you will
 have the Tangent CE; then adding together
 the Squares of AC and CE, and Product (by
 the 47.) will give the Square of AE: extract-
 ing therefore from that Number the Square
 Root, you will know the Length of the Line
 AE, which is the Secant.



By

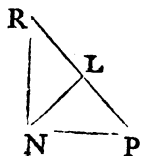


By this also we may augment Figures as much as we please. For Example: To double the Square ABCD, continue the Side CD, so that AD and DE may be equal; the Square of AE will be double the Square ABCD; since (*by the 47.*) it is equal to both the Squares of AD and DE. Making the right Angle AEF, and taking EF equal to AB, the Square of AF will be triple the Square ABCD. Again, Making the right Angle AFG, and taking FG equal to AB, the Square of AG will be quadruple, or four times the Square of ABCD. And that which I say of the Square, may be understood of all Similar Figures.

PROPOSITION XLVIII.

A THEOREM.

If in a Triangle the Square of one Side be equal to the Squares of both the other Sides, taken together; the Angle opposite to that first Side will be a Right Angle.



IF the Square of the Side NP be equal to both the Squares of the Sides NL and LP, taken together; the Angle NLP will be a right Angle. Draw LR perpendicular to NL,

NL, and equal to LP; then draw the Line NR.

Demonstration.

In the Rectangular Triangle NLP, the Square of NR is equal to the Squares NL, and RL, or LP, (*by the 47.*) Now the Square of NP is also equal to the same Squares of NL and LP; therefore the Square of NR is equal to the Square of NP, and by consequence the Lines NR and NP are equal. And because the Triangles, NLR and NLP, have the Side NL common; the Sides LP and LR equal, and their Bases NP and NR also equal; the Angles NLP and NLR will be equal, (*by the 8.*) then the Angle NLR being a right Angle, the Angle NLP must be so too.

T H E

THE
Geometric Algebra
 SECOND BOOK
 OF THE
 Elements of *EUCLID*.

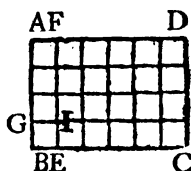
EUCLID in this Book treats of the Powers of right Lines; that is to say, of their Squares; comparing the divers Rectangles, which are made upon a Line divided, as well with the Square, as the Rectangle, of the whole Line. 'Tis a Part exceeding useful, serving for the Foundation of the principal Operations of *Algebra*. The three first Propositions demonstrate the third Rule or Operation of *Aritbmetick*, viz. *Multiplication*. The Fourth teaches to extract the Square Root of any Number whatsoever. Those that follow to the eighth, serve upon many Occasions in *Algebra*. The rest instruct us in Operations proper for *Trigonometry*. This Book seems at first View very difficult; because Men are apt to imagine there is something mysterious contain'd therein; nevertheless

‘ nevertheless the greatest part of its Demon-
 ‘ strations are grounded on this most evident
 ‘ Principle, That the Whole is equal to all its
 ‘ Parts taken together. But it ought not to
 ‘ discourage any, if they should not at the first
 ‘ Attempt fully comprehend them.

DEFINITIONS.

1. *A Rectangular Parallelogram is compris’d under two Lines, that form a right Angle.*

‘ **O**bserve, that hencefor-
 ‘ ward, by a Rectangle,
 ‘ we shall intend such a Paral-
 ‘ lelogram whose Angles are
 ‘ all right Angles, distinguish-
 ‘ ing it by giving its Longitude
 ‘ and Latitude, naming two of its Sides, which
 ‘ contain one of its Angles, as the Lines AB
 ‘ and BC. For the Rectangle ABCD is compris’d
 ‘ under the Lines AB and BC; BC denoting its Longitude,
 ‘ and AB its Latitude;
 ‘ and the other Sides being equal to these, it
 ‘ will not be necessary to name them. I have
 ‘ also formerly intimated, that the Line AB,
 ‘ remaining perpendicular to BC, and being
 ‘ mov’d from one Extremity thereof to the other,
 ‘ produces the Rectangle ABCD; and that that
 ‘ Motion has some resemblance to *Arithmetical*
 ‘ Multiplication; so that, as the Line A B
 ‘ moving



‘ moving over the Line BC, that is, taken so
 ‘ many times as there are Points in BC, com-
 ‘ poses the Rectangle ABCD; so the Multi-
 ‘ plication also of AB by BC, will give the
 ‘ Rectangle ABCD. As, Suppose I knew the
 ‘ Number of *Mathematical Points*, that are in
 ‘ AB, for Example 40, and that there were 60
 ‘ in BC: It is evident that the Rectangle
 ‘ ABCD will have so many Lines equal to AB,
 ‘ as there are Points in BC; and that multiplying
 ‘ 40 by 60, the Product will be 2400, which is
 ‘ the Number of *Mathematical Points* in the
 ‘ Rectangle A B C D.

‘ I may take what Quantity I please for a *Ma-*
 ‘ *thematical Point*, provided I do not afterwards
 ‘ subdivide it; it must therefore be observ’d,
 ‘ that when I measure a Line, for a *Mathemati-*
 ‘ *cal Point*, I take that Measure which best suits
 ‘ with my Occasions; for Example, when I say
 ‘ a Line of five Foot in Length, my *Mathema-*
 ‘ *tical Point* is a Line of a Foot long; which I
 ‘ take without considering that it is compos’d
 ‘ of any Parts. In measuring a Superficies
 ‘ likewise I do the same, taking some known
 ‘ Superficies; for Example, a Foot Square;
 ‘ which I do not afterwards subdivide. I make
 ‘ use of a Square rather than any other Figure;
 ‘ because its Length and Breadth being equal;
 ‘ there is no need of naming more than one of
 ‘ its Dimensions to describe it. Accordingly
 ‘ when I would mark the *Area* of the Rectangle
 ‘ ABCD, I do not consider the Sides as simple
 ‘ Lines

‘ Lines, but as Rectangles of a determinate
 ‘ Breadth: For Example; when I say that the
 ‘ Rectangle ABCD has the Side AB of four
 ‘ Foot long, since a Foot is to me instead of a
 ‘ *Mathematical Point*, I conceive the Side AB to
 ‘ have also a Foot in Breadth, and to be as the
 ‘ Rectangle ABEF. Therefore knowing how
 ‘ many times the Breadth BE is contain’d in the
 ‘ Line BC, I shall know how many times the
 ‘ Line AB is contain’d in the Rectangle ABCD;
 ‘ that is to say, multiplying A B (which has
 ‘ four Foot Square) by 6, the Product will be
 ‘ 24 Foot Square. In like manner, knowing
 ‘ the Magnitude of the Rectangle ABCD to be
 ‘ 24 Foot Square, and one of its Sides AB to
 ‘ be 4; dividing 24 by 4, the Quotient will
 ‘ give me the other Side BC, consisting of six
 ‘ Foot Square.

2. ‘ Having drawn the Diameter of A ED
 ‘ a Rectangle, one of the lesser Rect-
 ‘ angles, thro’ which it passes, toge-
 ‘ ther with the two Complements, is
 ‘ call’d the Gnomon. As the Rect-
 ‘ angle EG, through which the Diameter BD
 ‘ passes, together with the Complements EF and
 ‘ GH, is call’d the Gnomon; their Figure toge-
 ‘ ther representing a Carpenter’s Square.



PROPOSITION. I.

A THEOREM.

If two Lines be propos'd, whereof one is divided into divers Parts; the Rectangle contain'd under those two Lines is equal to the Rectangles contain'd under the Line which is not divided, and the Parts of the Line divided.

CG H D



A E F B

LET the Lines propos'd be AB, and AC; and let AB be divided into as many Parts as you please. The Rectangle AD contain'd under the Lines AB and AC, is equal to the Rectangle AG contain'd under AC and AE; to the Rectangle EH contain'd under EG equal to AC, and EF; and to the Rectangle FD contain'd under FH equal to AC, and FB taken together.

Demonstration.

The Rectangle AD is equal to all its Parts taken together; which are the Rectangles AG, EH, and FD, and no other. Therefore the Rectangle AD is equal to the Rectangles AG, EH and FD taken together.

By

By Numbers.

This Proposition holds true likewise in Numbers. Suppose the Line AC to be five Foot long; AE two, EF four, FB three; and by consequence AB nine: The Rectangle contain'd under AC five, and AB nine, that is to say, five times nine, which makes forty five, is equal to twice five, or ten, four times five, or twenty; and three times five, or fifteen; for ten, twenty, and fifteen, make forty five.

The USE.

By this Proposition is demonstrated the ordinary Operation of Multiplication. For Example; If you were to multiply the Number A, which is 53, by the Number B, that is 8. Divide the Number A into so many Parts as there are Characters: that is, two, 50, and 3; which multiply by 8, saying, eight times three is twenty four; and so you make one Rectangle. Then multiplying the Number 50 by 8, the Product will be 400. But 'tis evident, that the Product of eight times 53, being 424, is equal to the Product 24, and the Product 400 taken together.

A,	53
B,	8
C, 50,	3
B,	8
D,	24
E,	400
F,	424

PROPOSITION II.

A THEOREM.

The Square of any Line is equal to the Rectangles contained under the whole Line, and all its Parts.

C.GHD



A E F B

LET the Line propos'd be AB, and its Square ABDC. I say, the Square ABDC is equal to the Rectangles contain'd under the whole Line AB and AE; another under AB and EF; and a third under AB and FB taken together.

Demonstration.

The Square A B D C is equal to all its Parts taken together, which are the Rectangles AG, EH, FD. The first AG is contain'd under AC equal to AB, and AE. The second EH is contain'd under EG equal to AC or AB, and FE. The third FD is contain'd under FH equal to AB, and FB: But 'tis the same thing to be contain'd under a Line equal to AB, and to be contain'd under AB itself. Therefore the Square of AB is equal to the Rectangles contain'd under AB, and AE, EF, FB, the Parts of AB.

By Numbers.

Let the Line AB represent the Number 9; its Square will be 81. Let also the Part AE be four; EF three; and FB two; nine times four
make

makes thirty six; nine times three, twenty seven; and nine times two, eighteen; and 'tis plain that 36, 27, and 18 make 81.

The USE.

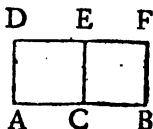
' This Proposition serves likewise to prove Multiplication; as also for Equations in *Algebra*.

PROPOSITION III.

A THEOREM.

If a Line be divided into two Parts, the Rectangle contained under the whole Line, and one of its Parts, is equal to the Square of the same Part, and the Rectangle contain'd under both the Parts.

LET the Line AB be divided into two Parts at the Point C; and let a Rectangle be made of the whole AB and one of its Parts AC; that is to say, Let AD be equal to AC; and then if the Rectangle AF be compleated, it will be equal to the Square of AC, and the Rectangle contain'd under AC and CB. Draw the Perpendicular CE.



Demonstration.

The Rectangle AF contain'd under AB and AD equal to AC, is equal to all its Parts, which are the Rectangles AE and CF. The first AE is the Square of AC, the Lines AC and AD being equal; and the Rectangle CF is contain'd under CB, and CE equal to AD, or AC. Therefore the Rectangle contain'd under AB and AC, is equal to the Square of AC, and the Rectangle contain'd under AC and CB.

By Numbers.

Let AB be 8; AC 3; and CB 5; the Rectangle contain'd under AB and AC, will be three times eight, or 24: The Square of AC 3, is nine; and the Rectangle contain'd under AC 3, and CB 5, is three times 5 or 15. But it is evident that 15 and 9 make 24.

The USE.

A,	43
C,	40, 3
B,	3
120.	9.
129	

' The Use of this Proposition is
 ' still to demonstrate the ordinary
 ' Practice of Multiplication. For
 ' Example; If you would multiply
 ' the Number 43 by 4, having divid-
 ' ed the Number 43 into 40, and 3;
 ' Three times 43 will amount to as
 ' many as three times three, or nine, that is
 ' the Square of three; and three times 40, that
 ' is, 120; for three times forty three is 129.
 ' Beginners ought not to be discourag'd, if they
 ' do

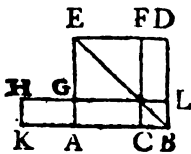
‘ do not presently apprehend these Propositions; which yet, in truth, are not difficult, but as they are conceiv’d to contain some strange Mystery.

PROPOSITION IV.

A THEOREM.

If a Line be divided into two Parts, the Square of the whole Line will be equal to the Squares of both the Parts; and two Rectangles contain’d under the same Parts.

LET the Line AB be divided in C, and its Square ABDE describ’d; let its Diagonal also EB be drawn, and a Perpendicular cutting it CF: And by that Point let the Line GL be drawn parallel to AB. ’Tis evident that the Square ABDE is equal to the four Rectangles GF, CL, CG, and LF. The two first of which are the Squares of AC and CB: And the two Complements are contain’d under AC and CB.



Demonstration.

The Sides AE and AB are equal; therefore the Angles AEB, and ABE are half right Angles: And because the Lines GL and AB are Parallels, the Angles of the Triangles of the Square GF (by the 29. 1.) will be equal, as also their

their Sides (*by the 6. 1.*) Therefore GF is the Square of AC. In like manner CL is the Square of CB : The Rectangle GC is contain'd under AC, and AG equal to BL or BC ; and the Rectangle LF is contain'd under LD equal to AC, and FD equal to BC.

Coroll. If you draw the Diagonal of a Square, the Rectangles which it cuts are Squares.

Demonstration.

A,	144
B,	22
C,	12

‘ This Proposition teaches the
 ‘ Method of extracting the Square
 ‘ Root of any Number propos’d. Let
 ‘ the Number be A, or 144 represent-
 ‘ ed by the Square AD, and its Root by the
 ‘ Line AB. I suppose it known from other
 ‘ Principles that it requires two Characters. I
 ‘ imagine therefore that the Line AB is divided
 ‘ in C, so that AC may represent the first
 ‘ Character, and BC the second. Then search-
 ‘ ing the Root of the first Character, of the
 ‘ Number 144, which is 100, I find it to be
 ‘ 10; and making its Square 100, represented
 ‘ by the Square GF, I subtract it from 144;
 ‘ and there remains 44 for the Rectangles GC.
 ‘ FL, and the Square CL : But because the
 ‘ Figure of a Gnomon is not proper for this
 ‘ Operation, I transport the Rectangle FL un-
 ‘ to KG, making one whole Rectangle KL, that
 ‘ is 44. I know also already almost the whole
 Side

‘ Side KB: For AC being 10, KC must be 20;
‘ I must therefore divide 44 by 20; that is to
‘ say, for my Divisor, doubling the Root found;
‘ I enquire then how many times 20 I can have
‘ in 44? And find twice; and therefore take 2
‘ for the Side BL; and because 20 was not the
‘ intire Side KB, but only KC; that two
‘ which came in the Quotient, I add to the Di-
‘ visor, making it 22; which Number being
‘ found precisely twice in 44, adding 2 to the
‘ Root before found, I conclude the whole
‘ Square Root of 144 to be 12. You see then
‘ that the Square 144 is equal to the Square of
‘ 10, which is 100, is Square of 2, that is 4;
‘ and twice 20, which make the two Rectan-
‘ gles contain’d under two and ten.

P R O

Rectangle AH, with the Square LG, will be equal to the Square CF.

By Numbers.

Let AB be 10; AC will be 5, and CB likewise; and let CD be 2, and BD 3, the Rectangle contain'd under AD 7, and DB 3, that is to say 21, with the Square of CD 2, that is 4, will be equal to the Square of CD 5, which is 25.

The U S R.

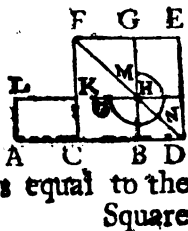
This Proposition is very useful in the third Book: It is also us'd in *Algebra*, to demonstrate the manner of finding the Root of an affected or impure Square.

PROPOSITION VI.

A THEOREM.

If a Line be divided into two equal Parts, and to it another Line added; the Rectangle contain'd under the Line compounded of those two, and that which is added, with the Square of half the divided Line, is equal to the Square of the Line compounded of that half, and the Line that is added.

IF to the Line AB, divided into two equal Parts in C, be added the Line BD; the Rectangle AN, contain'd under the Line AD, and DN equal to BD, with the Square of CB, is equal to the



Square of CD. Make the Square of CD, and having drawn the Diagonal FD, draw also BG parallel to FC, cutting FD at the Point H, through which passes the Line HN parallel to AD; KG will be the Square of CB; and BN, that of BD.

Demonstration.

The Rectangles AK and CH, being upon equal Bases AC and CB, are equal, (*by the 36. 1.*) The Complements CH and HE are equal, (*by the 43. 1.*) therefore the Rectangles AK and HE are equal. Add to both the Rectangle CN, and the Square KG: then the Rectangles AK and CN, that is, the Rectangle AN, with the Square KG, will be equal to the Rectangles CN and HE and the Square KG, that is, by 4. 2. the Square CE.

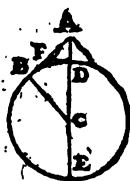
By Numbers.

Let AB consist of 8 Parts; AC of 4; and CB of 4; BD of 3; so that the whole AD be 11. 'Tis evident the Rectangle AN is three times 11, that is 33; which with the Square of KG, (equal to CB 4.) that is 16, makes 49, and therefore is equal to the Square of CD 7, which is 49; for 7 times 7 make 49.

The USE.

' *Maurolychus*, by the help of this Proposition, measur'd the whole Earth at one single Observation. To effect which, he advises, that

that from the Top of a Mountain of a known Height A , you observe the Angle BAC , made by the Line AB , touching the Superficies of the Earth at B , and the Line AC passing thro' the Centre; And that in the Triangle ADF , knowing the Angle A , and the right Angle ADF , you find, by *Trigonometry*, the Sides AF and FD : And because 'tis easie to demonstrate that FB and FD are equal, you will then know the Line AB , and also its Square. Now we have demonstrated in the preceding Proposition, that the Line ED , being divided into equal Parts in C , and the Line AD added to it; the Rectangle contain'd under EA and AD , with the Square of CD , and CB , is equal to the Square of CA ; and the Angle ABC , being a right Angle, (as is prov'd in the third Book) the Square of CA is equal to the Squares of AB and BC ; therefore the Rectangle under AE and AD , with the Square of BC , is equal to the Squares of AB and BC . Take therefore from them both the Square of BC , and the Rectangle under AE and AD will be equal to the Square of AB . Divide therefore the known Square of AB , by the Height of the Mountain AD , and the Quotient will be the Line AE ; from which subtracting the Height of the

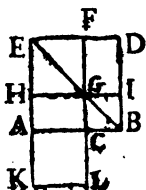


- ' the Mountain, the Remainder will be the Dia-
 ' meter of the Earth DE.
 ' We have made use likewise of the same Pro-
 ' position in our *Algebra*, to demonstrate the
 ' thirteenth Proposition of the third Book, to
 ' find the Root of a Square equal to a Number
 ' more a certain Number of Roots. The two
 ' that follow do also serve for the Proof of the
 ' like Operations.

PROPOSITION VII.

A PROBLEM.

If a Line be divided, the Square of the whole Line with that of one of its Parts, is equal to two Rectangles contain'd under the whole Line, and that first Part, together with the Square of the other Part.



LET the Line AB be divided
 any where in C; the Square
 AD of the Line AB, with the
 Square AL, will be equal to two
 Rectangles contain'd under AB
 and AC, with the Square of CB.
 Make the Square of AB, and having drawn the
 Diagonal EB, and the Lines CF and HGI; pro-
 long EA so far, as that AK may be equal to
 AC; so AL will be the Square of AC, and HK
 will be equal to AB; for HA is equal to GC,
 and

and GC, is equal to CB, because CI is the Square of CB (*by the Coroll. of the 4.*)

Demonstration.

'Tis evident, that the Squares AD and AL are equal to the Rectangles HL and HD, and the Square CI. Now the Rectangle HL is contain'd under HK equal to AB, and KL equal to AC. In like manner the Rectangle HD is contain'd under HI equal to AB, and HE equal to AC. Therefore the Squares of AB and AC are equal to two Rectangles contain'd under AB and AC, and the Square of CB.

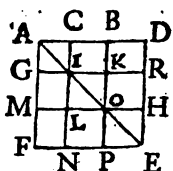
In Numbers.

Suppose the Line AB to consist of 9 Parts, AC of 4, and CB of 5. The Square of AB 9 is 81, and that of AC 4 is 16; which 81 and 16 added together make 97. Now one Rectangle under AB and AC, or four times 9, make 36, which taken twice, is 72; and the Square of CB 5 is 25; which 72 and 25 added together make also 97.

PROPOSITION VIII.

A THEOREM.

If you divide a Line, and add another to it equal to one of its Parts, the Square of the whole compounded Line will be equal to four Rectangles contained under the first Line, and that part that is added, together with the Square of the other Part.



LET the Line AB be divided any where in the Point C, and BD equal to CB added to it: the Square of AD will be equal to four Rectangles contain'd under AB, and BC or BD, and the Square of AC. Make the Square of AD, and having drawn the Diagonal AE, draw likewise the Perpendiculars BP and CN, cutting the Diagonal in O and I, and also the Lines MOH and GIR parallel to AB. The Rectangles GC, LK, PH, MB, and NR, will be Squares, (*by the Coroll. of the 4.*)

Demonstration:

The Square ADEF is equal to all its Parts; and the Rectangles LB, OD, PM are contain'd under Lines equal to AB and BC; and if you add the Rectangle MI to the Rectangle PH, they together will give you another Rectangle contain'd under a Line equal to AB, and another equal

equal to CA or BD. Besides which, there remains nothing but the Square GC, which is the Square of AC. Therefore the Square AD is equal to four Rectangles contain'd under AB and BD, and the Square of AC.

In Numbers.

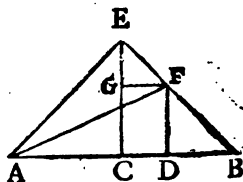
Let the Line AB consist of 7 Parts, AC of 3, and CB of 4, as also BD. The Square of AD 11 will be 121. And one Rectangle under AB 7, and BD 4, makes 28; which taken four times, is 112; and those together with the Square of 3, which is 9, make also 121.

PROPOSITION IX.

A THEOREM.

If a Line be divided into two equal Parts, and two unequal, the Squares of the unequal Parts will double the Square of half the Line, and the Square of the intermediate Part.

LET the Line AB be divided into two equal Parts at the Point C, and two unequal at the Point D; the Squares of the unequal Parts AD and DB will be double the Squares of AC, which is half AB, and CD the intermediate Part. Draw CE perpendicular to AB, and equal to AC; draw also



also the Lines AE and BE, and the Perpendicular DF, as likewise FG parallel to CD. Then join the Line AF.

Demonstration.

The Lines AC and CE are equal, and the Angle C is a right Angle: Therefore (*by the 5. 1.*) the Angles CAE and CEA are equal, and consequently half right Angles. In like manner, the Angles CEB, CBF, GFE, and DFB, are half right Angles; and the Lines GF and GE, DF and DB, equal, (*by the 6. 1.*) and the whole Angle AEF is a right Angle. Now the Square of AE (*by the 47. 1.*) is equal to the Squares of AC and CE, which are equal: Therefore it is double the Square of AC. For the same Reason, the Square of EF is double the Square of GF, or CD. Now the Square of AF is equal to the Squares of AE and EF, because the Angle AEF is a Right Angle; therefore the Square of AF is double the Squares of AC and CD. The same Square of AF is likewise equal to the Squares of AD, and DF or DB, the Angle D being a right Angle. Therefore the Squares of AD and DB are double the Squares of AC and CD.

In Numbers.

Let AB be 10, AC 5, CD 3, and DB 2; the Squares of AD 8, and DB 2, that is to say, 64 and 4, which make 68, are double the Squares of AC 5, that is, 25; and of CD 3, which
is

is 9; for 25 and 9 make 34, which is half of 68.

The USE.

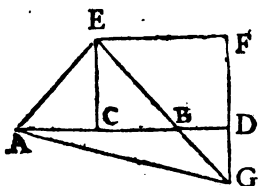
‘ I have not met with this Proposition, except in *Algebra*; no more than that which follows.

PROPOSITION X.

A THEOREM.

If a Line be added to another that is divided into two equal Parts; the Square of the Line compounded of those two, with the Square of that which is added, makes double the Square of half the Line, and the Square of that which is compounded of half, and the Line that is added.

LET the Line AB be divided in the middle at the Point C, and the Line BD added to it: The Squares of AD and BD will be double the Squares of AC and CD. Draw the Perpendiculars CE and DF equal to AC; and then draw the Lines AE, EF, and producing FB to G, so that DG may be equal to BD, join the Lines AG and EBG.



Demonstration.

The Lines AC, CB, and CE, being equal, and the Angles at the Point C being right Angles; the Angles CAE, AEC, CEB, and CBE, will be half right Angles. In like manner, the Angle D being a right Angle, and the Lines BD and DG equal, the Angles DBG and DGB will be half right Angles; and so will likewise GFF, the Angle F being a right Angle; therefore the Lines FG and FE are equal, (*by the 6. 1.*) and EF is equal to CD, (*by the 33. 1.*) Now the Square of AE is double the Square of AC, and the Square of EG is also double the Square of EF, or CD, (*by the 47. 1.*) But the Square of AG is equal to the Squares of AE and EG, (*by the same*;) therefore the Square of AG is double the Squares of AC and CD. The same Square of AG is likewise (*by the same 47.*) equal to the Squares of AD, and DG equal to DB: therefore the Squares of AD and DB are double the Squares of AC and CD.

By Numbers.

Let AB contain 6 Part, AC 3, and CB 3, BD 4; the Square of AD 10 is 100; the Square of BD is 16, which make together 116. The Square also of AC 3 is 9: The Square of CD 7 is 49. Now 49 and 9 make 58, the half of 116.

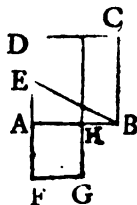
P R O-

PROPOSITION. XI.

A PROBLEM.

*To divide a Line in such a manner, that the Rect-
angle under the whole Line, and one of its Parts,
shall be equal to the Square of the other Part.*

Suppose the Line AB to be divided in such a manner that the Rectangle under the whole Line AB and BH, may be equal to the Square of AH. Make the Square of AB (by the 46. 1.) and dividing AD in the Middle E, draw EB, and take EF equal to EB. Then make the Square of AF, that is to say, let AF and AH be equal. I say, the Square of AH will be equal to the Rectangle HC, contain'd under HB, and BC equal to AB



Demonstration.

The Line AD is divided equally in the Point F, and the Line FA added to it; therefore (by the 6.) the Rectangle DG contain'd under DF, and FG equal to AF, with the Square of AE, is equal to the Square of EF, equal to EB: now the Square of EB is equal to the Squares of AE and AB, (by the 47. 1.) Therefore the Squares of AB and AE are equal to the Rectangle DG, and the Square of AE; and subtracting from both the Square of AE; the Square of AB, that is, AC, will be equal to the Rectangle DG: Taking away therefore the Rectangle DH,

H 3

which

which is common to both, the Rectangle HC will be equal to the Square of AH, that is, AG.

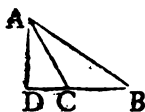
The USE.

‘ This Proposition teaches how to cut a Line according to the extreme and middle Proportion, as will be shewn in the sixth Book. ’Tis also frequently made Use of in the fourteenth Book of *Euclid’s Elements*, to find the Sides of Regular Solids. It is useful also in the 11. of the 4. to inscribe a Pentagon in a Circle, as also a Pentedecagon (or a Figure with 15 Angles.) You will see also other Uses thereof in dividing Lines after this manner, in the 30th Proposition of the 6.

PROPOSITION XI.

A THEOREM.

In an Obtuse-angled Triangle, the Square of the Side oppos’d to the Obtuse Angle, is equal to the Squares of both the other Sides, and two Rectangles contain’d under the Line upon which a Perpendicular will fall, and the Line which lies betwixt the Triangle and the Perpendicular.



LET the Angle ACB of the Triangle ABC be obtuse, and let AD be drawn Perpendicular to BC; the Square of the Side AB is equal to the Squares of the Sides AC and CB, and two Rectangles contain’d under the Side CB and CD.

Demon-

Demonstration.

The Square of AB is equal to the Squares of AD and DB, (*by the 47. 1.*) But the Square of BD is equal to the Squares of DC and CB, and two Rectangles contain'd under DC and CB (*by the 4.*) Therefore the Square of AB is equal to the Squares of AD, DC, and CB, and two Rectangles contain'd under DC and CB. Instead of the two first Squares of AD and DC, put the Square of AC, which is equal to them, (*by the 47. 1.*) then the Square of AB will be equal to the Squares of AC and CB, and two Rectangles contain'd under DC and CB.

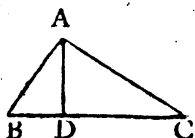
The USE.

' This Proposition is useful in *Trigonometry*,
' to measure the *Area* of a Triangle, whose
' Sides are known. For Example, Suppose the
' Side AB to consist of 20 Foot, AC of 13, BC
' of 11; the Square of AB will be 400, that of
' AC 169, and that of BC 121. The Sum of
' the two last is 290, which subtracted from
' 400, there will remain 110 for the two Rectan-
' gles under BC and CD. The half of which,
' 55, will make one half of those Rectangles;
' dividing which Number by BC, 11, we shall
' have 5 for the Line CD; whose Square 25 be-
' ing subtracted from the Square of AC, 169,
' leaves the Square of AD, 144, whose Root
' 12 will be the Side AD; which being multi-
' plied by 5, the half of BC, will give the
' *Area* of the Triangle ABC, that is, 66 Foot
' Square.

PROPOSITION XIII.

A THEOREM.

In any Triangle whatsoever, the Square of the Side oppos'd to the acute Angle, with two Rectangles contain'd under the Side upon which the Perpendicular falls, and the Line which is betwixt the Perpendicular and that Angle, is equal to the Squares of both the other Sides.



Suppose the Triangle to be ABC, and the acute Angle C, and AD the Perpendicular falling upon BC; the Square of the Side AB, oppos'd to the acute Angle C, with two Rectangles contain'd under BC and CD, will be equal to the Square of AC and BC.

Demonstration.

The Line BC being divided in D, (*by the 7.*) the Squares of BC and DC are equal to two Rectangles under BC and CD, and the Square of BD. Adding to both the Square of AD; the Squares of BC, DC, and AD, will be equal to two Rectangles under BC, and CD, and the Squares of BD, and AD. Instead of the Squares of CD, and AD, put the Square of AC, which is equal to them, (*by the 47. 1.*) and instead of the Squares of BD, and AD, substitute the Square of AB, which is equal to them (*by the same*;) then the Squares of BC and AC will be equal

equal to the Square of AB, and two Rectangles contain'd under BC and CD.

The USE.

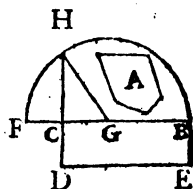
‘ These Propositions are very useful in *Trigonometry*: I have made use of them in the eighth Proposition of my third Book, to prove that in a Triangle the Sine Total has the same Proposition to the Sine of an Angle, as the Rectangle contain'd under the Sides, which form that Angle, to double the Triangle. I have us'd them likewise in the seventh, and divers other Propositions.

PROPOSITION XIV.

A PROBLEM.

To describe a Square equal to a Rectilineal Figure given.

TO describe a Square equal to the Rectilineal A: make (*by the 45. 1.*) a Rectangle BCDE equal to the Rectilineal A. If the Sides CD and CB were equal, we should have what we desir'd: But being unequal, continue the Line BC, so that CF may be equal to CD: and dividing the Line BF in the middle at the Point G, describe the Semi-circle FHB; this done, prolong DC to H. The Square of CH is equal to the Rectilineal A. Draw the Line GH.



Demon-

Demonstration.

The Line BF is divided into two equal Parts in G, and two unequal in C; therefore (*by the 5.*) the Rectangle contain'd under BC, CF, or CD; that is to say, The Rectangle BD, with the Square of CG, is equal to the Square of GB, or GH, which equal to it. Now (*by the 47. 1.*) the Square of GH is equal to the Squares of CG, and CH: Therefore the Rectangle BD, and the Square of CG, are equal to the Squares of CG, and CH; and therefore, taking away the Square of CG, which is common to both, there will remain the Square of CH equal to the Rectangle BD, or, which is the same, the Rectilineal A.

The USE.

‘ This Proposition teaches us, in the first
 ‘ Place, to reduce any Rectilineal Figures to
 ‘ Squares; which being the chief Measure of
 ‘ all Superficies, because its Dimensions are
 ‘ both equal, we can by this means take the
 ‘ Magnitude of all sorts of Rectilineal Figures.
 ‘ Again, it helps us to find a middle Proportional
 ‘ betwixt two Lines given, as we shall
 ‘ see in the thirteenth Proposition of the sixth
 ‘ Book.

‘ *Aristotle* brings this Proposition as an Instance of a formal Definition: For, in his 2d
 ‘ Book, *de Anima*, § 12. distinguishing betwixt
 ‘ a formal and a causal Definition, he explains
 ‘ them thus: If when ’tis demanded, what it is
 ‘ to

‘ to square a Rectangle? Answer be return’d,
‘ That it is to describe a Square equal to a Rectilineal given; this Answer contains the formal Definition. But if it be said, That it is
‘ to find a middle Proportional betwixt two
‘ Lines, this gives the causal Definition. For
‘ to find a middle Proportional, is the Cause of
‘ making a Square equal to the Rectilineal propos’d.

‘ This Proposition may also be farther useful
‘ for the squaring of crooked Figures, and also,
‘ as far as is possible, even the Circle itself;
‘ for all sorts of crooked Figures may, at least
‘ as far as is discernable by Sense, be reduc’d to
‘ Rectilineals. As for Example: if we inscribe
‘ in a Circle a Polygon consisting of a thousand
‘ Sides, there will be no sensible difference betwixt it and the Circle; therefore reducing
‘ this Polygon to a Square, we do, as far as
‘ our Senses are capable of judging, square the
‘ Circle.

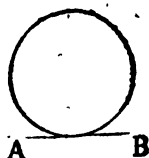
THE
THIRD BOOK
OF THE
Elements of *EUCLID*.

‘ **T** HIS *Third Book* explains the Pro-
 ‘ perties of a Circle, and compares
 ‘ divers Lines which may be drawn
 ‘ within, or without, its Circumference. It
 ‘ considers likewise the Circumstances of Circles,
 ‘ that cut each other, or touch a right Line;
 ‘ and the Differences of Angles that are made
 ‘ either at the Centres, or Circumferences. In
 ‘ fine, it lays down the first Principles for esta-
 ‘ blishing the Practical Part of *Geometry*; for
 ‘ which the Circle is most commodiously made
 ‘ Use of in almost all Treatises of the *Mathe-*
 ‘ *matics*.

DEFI-

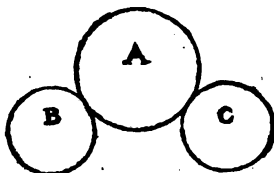
DEFINITIONS.

1. Those Circles are equal, whose Diameters or Semi-diameters are equal.

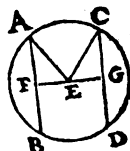


2. A Line is said to touch a Circle, when, meeting with its Circumference, it does not cut it: As the Line AB.

3. Circles touch, when meeting, they do not cut each other: As the Circles A, B, and C.

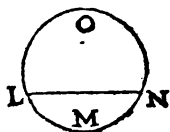


4. Those Lines are equally remote from the Centre; when the Perpendiculars, drawn from the Centre to the Lines, are equal.



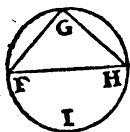
‘ As for Example; If EF, and EG, Perpendiculars to the Lines AB, and CD be equal, AB and CD will be equally remote from the Centre; because the Distance ought always to be measur’d by perpendicular Lines.

5. A Segment of a Circle is a Figure terminated on one Side by a right Line, and on the other by the Circumference of a Circle: As L O N, L M N.



6. The

6. The Angle of the Segment is the Angle which the Circumference makes with the right Line : As the Angles LNG, NLM.



7. An Angle is in that Segment in which are the Lines that form it : As the Angle FGH, is in the Segment FGH.

8. An Angle is upon that Arch, to which it is oppos'd, or which is as its Base : As the Angle FGH, is upon the Arch FIH.

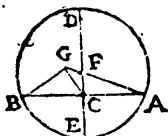


9. The Sector is a Figure contain'd under two Semi-diameters, and the Arch which serves them for a Base : As the Figure FIGH.

PROPOSITION I.

A PROBLEM.

To find the Centre of a Circle.



TO find the Centre of the Circle AEBC, draw the Line AB, and divide it in the middle at the Point C; through which draw the Perpendicular ED, which also divide into two equal Parts at the Point F, then that Point F will be the Centre.

tre of the Circle. If it be not, suppose the Point G to be the Centre; and draw the Lines GA, GB, and GC.

Demonstration.

If the Point G be the Centre, the Triangles GAC and GBC will have the Sides GA and GB equal, (*by the Definition of a Circle:*) And AC and CB will be equal, the Line AB being divided in the middle at the Point C, and CG being common, the Angles GCB and GCA will be equal, (*by the 8. I.*) and CG a Perpendicular, not CD, which is contrary to the Supposition. Therefore the Centre must of necessity be in the Line CD. I add, That it must be at the Point F, where it is divided into two equal Parts; otherwise the Lines drawn from the Centre to the Circumference would not be equal.

Coroll. The Centre of a Circle is in that Line which falling perpendicularly upon another, divides it into two equal Parts.

The USE.

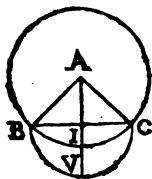
‘ This Proposition is necessary to demonstrate those that follow.

P R O-

PROPOSITION II.

A THEOREM.

A Right Line drawn from one Point of the Circumference to another, falls wholly within the Circle.



LET a Line be drawn from the Point B to the Point C: I say, it will be wholly contain'd within the Circle. To prove, that it cannot fall without the Circle, as BVC; having found the Centre of the Circle A, draw the Lines AB, AC, and AV.

Demonstration.

The Sides AB and AC, of the Triangle GBC, are equal : Therefore (*by the 5. 1.*) the Angles ABC and ACB are equal. And since the Angle AVC is an external Angle in respect of the Triangle AVB, it is greater than the Angle ABC (*by the 16. 1.*) and then also it will be greater than the Angle ACB. Therefore (*by the 19. 1.*) in the Triangle ACV, the Side AC, oppos'd to the greater Angle AVC, will be greater than AV; and by consequence AV ought not to reach to the Circumference of the Circle, if the Line BVC was a right Line.

The

The USE.

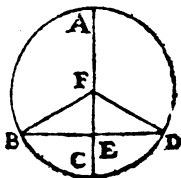
' 'Tis by this Proposition that they demon-
' strate, that a Circle can touch a right Line
' but in one Point. For if the Line touched
' two Points of the Circumference, it would
' be drawn from one of its Points to another:
' and by consequence, according to this Pro-
' position, would enter the Circle; though, by
' its Definition, the Line that touches ought
' not to cut the Circumference. *Theodosius*
' makes use of the same Demonstration to prove,
' that a Globe can touch a Plane only in one
' Point; for otherwise the Plane would enter
' within the Globe.

PROPOSITION III.

A THEOREM.

*If the Diameter divide a Line which does not pass
through the Centre into two equal Parts, it will
cut it at right Angles; and if it cut it at right
Angles, it will divide it into two equal Parts.*

IF the Diameter AC cut the
Line BD, which does not pass
thro' the Centre F, into two e-
qual Parts at the Point E, it will
cut it at right Angles. Draw
the Lines FB and FD,



I

Demon-

Demonstration.

In the Triangles FEB and FED, the Side EF is common; the Sides BE and ED are equal; because the Line BD is equally divided in E, and their Bases FB and FD are equal: Therefore (*by the 8. 1.*) the Angles BEF and DEF are equal, and by consequence right Angles.

I add, That if the Angles BEF and DEF be right Angles, the Line BD will be divided into two equal Parts at E, that is to say, the Lines BE and ED will be equal.

Demonstration.

The Triangles BEF and DEF are Rectangular: Therefore (*by the 47. 1.*) the Square of the Side DF will be equal to the Squares of the Sides ED and EF. Now the Squares of BF and FD are equal, because the Lines are equal; therefore the Squares of BE and EF are equal to the Squares of DE and EF; and taking away the Square of EF, which is common, the Squares of BE and ED will be equal, and by consequence the Lines.

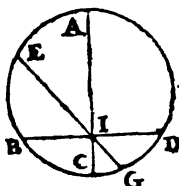
P R O-

PROPOSITION IV.

A THEOREM.

Two Lines drawn within a Circle, cannot cut each other into two equal Parts, unless they both pass through the Centre.

IF the Lines AC and BD cut each other at the Point I, which is not the Centre of the Circle, they will not equally divide each other. First, If one of those Lines, as AC, pass through the Centre, 'tis evident it cannot equally be divided but at the Centre. But if neither pass through the Centre, as BD and EG, draw the Line AIC through the Centre.



Demonstration.

If the Line AC divide the Line BD into two equal Parts in I, the Angles AID and AIB will be right Angles, (*by the 3.*) In like manner, if the Line EG was equally divided in I, the Angle AIE would be a right Angle; and consequently the Angle AIB and AIE would be equal, which is impossible, one being but part of the other. In a word, The Line AIC, which passes through the Centre, would be perpendicular to the Lines BD and EG, if they were both equally divided at the Point I.

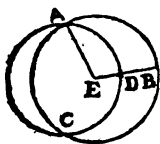
The USE.

‘ These two Propositions are us’d in *Trigonometry*, to demonstrate, That the Half of a Chord of an Arch is perpendicular to the Semi-diameter; and consequently, that it is the Sine of half the Arch. By these also they demonstrate, That the Sides of a Triangle have the same Proportion as the Sines of the opposite Angles. We also make Use of it to find the Eccentricity of the Circle, which the Sun describes in his *Annual Motion*.

PROPOSITION V.

A PROBLEM.

Circles that cut each other, have not the same Centre.



THE Circles ABC, and ACD, which cut each other in A and C, have not the same Centre. If they had the same Centre, suppose E, the Lines EA and ED would be equal, (*by the Definition of a Circle;*) as also the Lines EA and EB: Therefore the Lines ED and EB would be equal; which is impossible, one being part of the other.

P R O.

PROPOSITION VI.

A THEOREM.

Two Circles that touch each other on the inner Side, have not the same Centre.

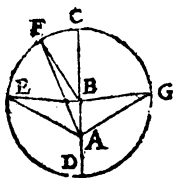
THE Circles BD and BC, which touch each other on the inner Side at the Point B, have not the same Centre. For should the Point A be suppos'd to be the Centre of both the Circles, the Lines AB and AC, AB and AD, would be equal, (*by the Definition of a Circle;*) and consequently the Lines AD and AC would be equal, which is impossible, one being part of the other.



PROPOSITION VII.

A THEOREM.

If many Lines be drawn from any one Point within the Circle, which is not its Centre, to the Circumference: 1. That which passes through the Centre is the greatest: 2. The Remainder of it, continu'd to the opposite part of the Circumference, is the least: 3. That which is nearest to the greatest, exceeds those that are more remote: 4. There can be no more than two of them equal to each other.



Suppose many Lines to be drawn from the Point A, being not the Centre of the Circle, to the Circumference; and the Line AC to pass through the Centre B: I will demonstrate, that it is greater than any of the other; for Example, that it is greater than AF. Draw FB.

Demonstration.

1. The Sides AB and BF of the Triangle ABF, are greater than AF alone, (*by the 20. 1.*) But BF and BC are equal, (*by the Definition of a Circle.*) Therefore AB and BC, that is to say, AC, is greater than AF.

2. I add, in the second Place, That AD is the least; for Example, that it is less than AE: Draw BE.

Demon-

Demonstration.

The Sides EA and AB are greater than BE alone, but BE is equal to BD; therefore AE and AB are greater than BD; taking therefore from both that which is common AB, AE will remain greater than AD.

3. Further, AF, which is nearer AC than AE, is also greater than AE.

Demonstration.

The Triangles FBA, and EBA, have the Sides BF and BE equal, and BA is common to both: But the Angle ABF is greater than the Angle ABE: Therefore, (*by the 24. 1.*) AF is greater than AE.

4. Lastly, I say, That no more than two Lines, that are equal to each other, can be drawn from the Point A to the Circumference. Take the Angles ABE and ABG equal; and draw the Lines AE and AG.

Demonstration.

The Triangles ABG and ABE, having the Sides BE and BG equal; the Side AB common to both, and the Angles ABE and ABG equal; therefore their Bases AE and AG will be equal, (*by the 4. 1.*) But all the Lines that can be drawn either on one Side or the other, will be either nearer AC, than AE and AG; or more remote from it; and accordingly will be either greater or less than AG. Therefore there can be no more than two Lines equal betwixt themselves, drawn from the Point A to the Circumference.

The USE.

‘ *Theodofus*, advantageously uses this Proposition to prove, That if from any Point of the Superficies of a Sphere, which is not the Pole of any certain Circle, divers Arches of greater Circles be drawn to the Circumference of that Circle, that which passes through its Pole will be the greatest. For Example; If from the Pole of the World, which is distinct from the Pole of the Horizon, (for the Zenith is its Pole,) divers Arches of greater Circles be drawn to the Circumference; the Arch of the Meridian, which passes through the Zenith, will be the greatest Arch. This Proposition is also brought to prove, that the Sun, when in his *Apogæum*, is most remote from the Earth.

P R O-

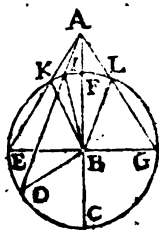
PROPOSITION VIII.

A THEOREM.

If from a Point taken without the Circle, many Lines be drawn to its Circumference : 1. Of all those that extend to the Concave Circumference, that which passes through the Centre is the greatest : 2. Those that lie nearest to it, are greater than those that are more remote : 3. Among those that fall upon the Convex Circumference, that which being continued passes through the Centre, is the least : 4. The nearer to that, are less than those farther off : 5. There can be but two equal Lines drawn from the same Point, either to the Concave or Convex Circumference.

Suppose many Lines were drawn from the Point A to the Circumference of the Circle GCDE.

First, The Line AC, which passes through the Centre B, is the greatest of all those that reach to the Concave Circumference ; for Example, it is greater than AD. Draw the Line BD.



Demonstration.

In the Triangle ABD, the Sides AB and BD are greater than AD alone ; but the Sides AB and

and BC are equal to AB and BD: Therefore AB and BC, or AC, is greater than AD.

2. AD is greater than AE.

Demonstration.

The Triangle ABD and ABE have the Side AB common to both, and the Sides BD and BE equal, and the Angle ABD is greater than the Angle ABE; therefore (*by the 24. 1.*) the Base AD is greater than the Base AE.

3. AF, which being continu'd, passes through the Centre, is the least of all those Lines that are drawn to the Convex Circumference LFIK; for Example, It is less than AI. Draw IB.

Demonstration.

In the Triangle AIB the Sides AI and IB are greater than AB alone, (*by the 24. 1.*) therefore taking, from both, the equal Lines BI and BF, AF will remain less than AI.

4. AI is less than AK. Draw the Line BK.

Demonstration.

In the Triangles AIB and AKB, the Sides AK and KB are greater than the Sides AI and IB, (*by the 21. 1.*) therefore taking, from both, the equal Sides BK and BI, AI will remain less than AK.

5. There can be but two Lines equal betwixt themselves drawn from A. Take the Angles ABL, and ABK; as also ABE, and ABG, equal.

Demonstration.

The Triangles ABL, and ABK, will have their Bases AL and AK equal, (*by the 4. 1.*) and by the same also AE and AG will be equal; but

but no other Line can be drawn, that will not be either nearer to, or more remote from AF, or AC; and consequently, that will not be either greater or less than AK and AL, AE and AG.

PROPOSITION IX.

A THEOREM.

That Point from whence three equal Lines can be drawn to the Circumference of a Circle, is its Centre.

IF the Point were not the Centre of a Circle, there could be but two equal Lines drawn from it to the Circumference, (by the 7, and 8.)

PROPOSITION X.

A THEOREM.

Two Circles cut each other only in two Points.

IF two Circles AEBD, and ABFD, should cut each other in three Points, A, B, and D; find (by the 1.) the Centre C of the Circle AEBD; and draw the Lines CA, CB, and CD.



Demon-

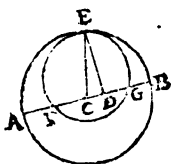
Demonstration.

The Lines, CA, CB, and CD, drawn from the Centre C to the Circumference of the Circle AEBD, are equal: But the same Lines are also drawn to the Circumference of the Circle ABFD: Therefore (*by the 9.*) the Point C will be the Centre of the Circle ABFD. So that two Circles which cut each other, will have the same Centre; which is contrary to the fifth Proposition.

PROPOSITION XI.

A THEOREM.

If two Circles touch each other on the Inside, a Line drawn through both the Centres, will also pass through the Point where they touch.



IF the two Circles EAB and EFG touch each other on the Inside, at the Point E; a Line drawn through both their Centres will pass through the Point E. For if the Point D were the Centre of the lesser Circle, and C that of the greater; so that the Line CD passing through both, should not pass through the Point E: Draw the Lines CE and DE.

Demonstration.

The Lines DE, and DG, drawn from the Centre of the lesser Circle D to its Circumference,

rence, would be equal: And adding the Line CD, the Lines ED and DC, would be equal to CG. Now ED and DC, are greater than EC alone, (*by the 20. 1.*) and so CG will be greater than CE: Yet C being the Centre of the greater Circle, CE and CB are equal; therefore CG will be greater than CB, which is impossible.

PROPOSITION XII.

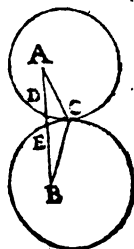
A THEOREM.

If two Circles touch each other on the Outside, a Line drawn through both their Centres, will pass through the Point where they touch.

IF the Line AB, which does not pass through the Point C where the Circles touch, be said to be drawn from the Centre A to the Centre B: Draw the Lines AC and BC.

Demonstration.

In the Triangle ACB, the Sides AC and BC would not be greater than the Side AB alone, (which is contrary to the 20. 1.) because AD and AC, as also BE and BC, are equal.

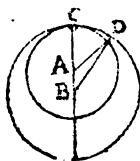


P R O-

PROPOSITION XIII.

A THEOREM.

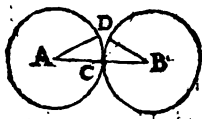
Two Circles can touch each other only in one Point,



First, If two Circles touch each other on the inside, they will touch but in one Point only, viz. the Point C, which is mark'd out by the Line BAC passing through both their Centres, A and B. For if they should touch likewise in the Point D, draw the Lines AD, and DB.

Demonstration.

The Lines AC and AD, drawn from the Centre of the lesser Circle to its Circumference, are equal; and adding AB, the Lines BA and AC, and BA and AD, would be equal. Now BC and BD, drawn from the Centre of the greater Circle to its Circumference, will be equal; therefore the Sides BA and AD will be equal to the Side BD alone, which is contrary to the 20. L.



Secondly, If two Circles touch each other on the outside; drawing the Line AB from one Centre to the other, it will pass through the Point C, where the Circles touch, (by the 12.) But if you say, That they touch also at the Point D; having drawn the Lines AD and BD; the Lines BC and BD, AC

AC and AD, being equal, the two Sides of a Triangle taken together, would be equal to the third, which is contrary to the 20. 1.

The USE.

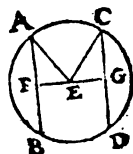
These four Propositions are very clear and evident, and also necessary in *Astronomy*, when we make Use of *Epycles*, to explain the Motions of the *Planets*.

PROPOSITION XIV.

A THEOREM.

Equal Lines drawn within a Circle, are equally remote from their Centre; and those that are equally remote from the Centre, are equal.

Supposing the Lines AB and CD to be equal: I will prove, that the Perpendiculars EF and EG, drawn from the Centre, are also equal. Draw the Lines EA and EC.



Demonstration.

The Perpendiculars EF and EG divide the Lines AB and CD in the middles at the Points F and G, (by the 3. 3.) therefore AF and CG are equal. The Angles F and G are right Angles: Therefore (by the 47. 1.) the Square of EA is equal to the Squares of EF and FA, as also the Square of EC is equal to the Squares of EG and GC: But the Squares of EA and EC are equal, because the Lines EA and EC are

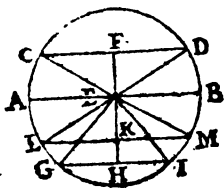
are equal: Therefore the Squares EF and FA are equal to the Squares EG and GC: And taking away the equal Squares AF and CG, there will remain the Squares of EF and EG equal; and consequently the Lines EF and EG, which are the Distances of the Lines AB and CD from the Centre, are equal.

But supposing the Distances, or Perpendiculars EF and EG to be equal; I will prove after the same manner, that the Squares of EF and FA are equal to the Squares of EG and GC; and taking away the equal Squares of EF and EG, there will remain the Squares of AF and CG equal. And therefore the Lines AF and CG, and their Doubles AB and CD, are equal.

PROPOSITION XV.

A THEOREM.

The Diameter is the greatest of all Lines inscrib'd in a Circle; and, of the rest, that is the greatest, which is nearest the Centre.



THE Diameter AB is the greatest of all Lines that can be drawn in the Circle GIDC: As for Example, it is greater than CD; for draw the Lines EC and ED.

Demon-

Demonstration.

In the Triangle CED, the Sides EC and ED are greater than CD alone, (*by the 20. 1.*) but AE and EB, or AB, is equal to EC and ED; therefore the Diameter AB is greater than CD.

Secondly, Let the Line GI be more remote from the Centre than the Line CD; that is to say, let the Perpendicular EH be greater than the Perpendicular EF. I say, That CD is greater than GI: Draw the Lines EC, and EG.

Demonstration.

The Squares of CF and FE (*by the 47. 1.*) are equal to the Square of EC: But the Square of EC is equal to the Square of EG, and the Square of EG equal to the Squares of GH and HE: Therefore the Squares of CF and FE are equal to the Squares of GH and HE; and taking from one Side the Square of HE, and from the other the Square of EF, which is less than the Square of HE, the Square CF will remain greater than the Square of GH: Therefore the Line CF will be greater than the Line GH; and the whole Line CD, the double of CF, will be greater than GI, the double of CH.

The USE.

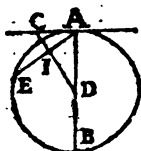
Theodofius makes Use of these two Propositions, to demonstrate, that in a Sphere the lesser Circles are more remote from the Centre. I have also made use of them in my Books of *Astronomical*. To these Propositions may likewise be referr'd that Mechanical Proposition of *Aristotle*, by which he shows, that the

Rowers at the middle of a Gally have greater force, than those that are at either the fore or hinder part thereof; because the Sides of the Gally being crooked, the Oars of the middle part are longer, *i. e.* reach farther than the rest. The Demonstrations relating to the *Iris*, or Rainbow, do also suppose the Truth of these Propositions.

PROPOSITION. XVI.

A THEOREM.

A Line drawn perpendicularly upon the Extremity of the Diameter, falls wholly on the Outside of the Circle, and touches it. But any other Line drawn betwixt that and the Circumference of the Circle, enters within the Circle, and outs it.



LET the Perpendicular AC be drawn upon the Point A, which is the Extremity of the Diameter AB: I say, First, That all the other parts of the same Line, for Example, the Point C, fall on the Outside of the Circle. Draw the Line DC.

Demonstration.

Since the Angle DAC of the Triangle DAC is a right Angle, DCA will be an Acute: And (by the 19. 1.) the Side DC will be greater than the Side DA; therefore the Line DC reaches beyond the Circumference of the Circle.

I add,

I add, That the Line CA touches the Circle; because that meeting with it at the Point A, it does not cut it, but all its Points are on the Outside of the Circle.

I say also, that no other Line can be drawn from the Point A below CA, which does not cut the Circle. If there could, suppose EA to be such an one; and from the Point D draw a Perpendicular to it, DI.

Demonstration.

Since the Angle DIA is a right Angle, and the Angle AID an Acute, AD will be greater than DI: Therefore the Line DI does not reach to the Circumference, but the Point I is within the Circle.

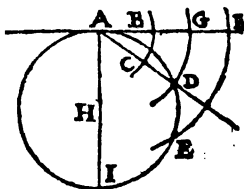
The U S E.

‘ Some Philosophers use this Proposition,
‘ but altogether in vain, to prove that Quan-
‘ tity is not divisible *in infinitum*, or that there
‘ really are in the World such things as *Zenoni-*
‘ *cal*, i. e. *absolutely and in their own nature in-*
‘ *divisible* Points. For the Proposition does not,
‘ as they would have it, prove, that a Circle
‘ touches a right Line in a *Zenonical*, but in a
‘ *Mathematical Point*, which is nothing else but
‘ a Quantity consider’d without Distinction of
‘ Parts, that is to say, without conceiving them
‘ distinct and separate one from the other;
‘ whether in Reality it has such Parts or not,
‘ making no Matter. We can therefore take
‘ any Quantity whatsoever for a *Mathematical*
‘ *Point*; which being once establish’d, our

' Circle will consist of such Points, and will
 ' be *Mathematically* perfect, provided it touch
 ' not a right Line, but in a part equal to that
 ' quantity which we have taken for a Point.
 ' But if we afterwards take a less part for our
 ' *Mathematical Point*, the Circle which was ex-
 ' actly perfect according to the first Suppositi-
 ' on, will be imperfect in the second, and de-
 ' generate into a Polygone. I believe, 'tis as
 ' impossible to describe a Circle, that accord-
 ' ing to any Supposition whatsoever shall be
 ' most exactly perfect, as it is to conceive the
 ' least possible Quantity.

' Secondly, Those Consequences, which some
 ' Men draw from this Proposition relating to
 ' the *Angle of Contact*, which they take to be
 ' less than any Rectilineal Angle, are ground-
 ' ed upon this Mistake, That they imagine an
 ' Angle to be a true Quantity; the contrary of
 ' which may appear from hence, That the
 ' Lines, that contain an Angle, being produc'd
 ' to any Length, the Angle becomes not at all
 ' the greater. Further, It ought to be duly
 ' consider'd, what we mean, when we say,
 ' that one Angle is greater than another; for
 ' this is all we understand, That a Circle be-
 ' ing describ'd from the Point of Concourse at
 ' any Distance whatsoever, the Lines of that we
 ' call the greater Angle will contain betwixt
 ' them a greater Arch of that Circle, than those
 ' of that we call the less; which is the sole
 ' meaning of the Excess of one Angle above
 ' another.

another. From whence I infer, that the Angle of Contact no more can be compar'd with a Rectilineal Angle, than a Superficies with a Line, being at the same time both equal and greater and less than a Rectilineal Angle. As for Example; From the Point A draw the Line AD, making with AE a Rectilineal Angle; I say, it is both greater and less than, and equal to, the Angle of Contact. For if we suppose divers Circles describ'd from the Point A, as the Centre, whereby to measure those Angles; it is evident that, according to the Arch drawn beyond the Point D, that is, the Arch EF, the Angle of Contact is greater than the Rectilineal Angle. But, on the contrary, according to the Arch CB, the Rectilineal Angle is the greater of the two. And lastly, according to the Arch DG, passing through the Point in which AD cuts the Circumference, they are both equal. From whence it follows, That the Angle of Contact is at the same time both less and greater than, and equal to, the Rectilineal Angle: And consequently, they ought not at all to be compar'd together. In a Word, Angles are no Quantities; nor are they call'd less or greater one than another, but with respect to the Arches which they contain: So that all

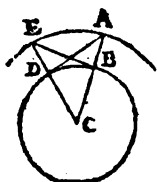


the Disputes about the Angle of Contact, and all the Paradoxes, conclude nothing either for or against the Divisibility of Quantity; an Angle being no Species, but only a Property thereof.

PROPOSITION XVII.

A PROBLEM.

From a Point given, to draw a Line that may touch a Circle.



TO draw a Line from the Point A touching the Circle BD, draw the Line AC to its Centre; and at the Point B draw a Perpendicular BE, which may cut an Arch of a Circle, describ'd from the Centre C through the Point A, at the Point E. Draw also the Lines EC, and AD. I say, The Line AD touches the Circle in D.

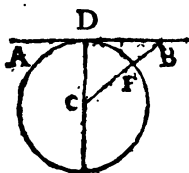
Demonstration.

The Triangles EBC and ADC have the same Angle C; and the Sides CD and CB, CE and CA, equal (*by the Defn. of a Circle* :) And therefore they are equal in all Respects, (*by the 4. 1.*) and the Angles CBE and CDA are equal. But the Angle CBE is a right Angle, therefore the Angle CDA will be so too, and (*by the 16. 3.*) the Line AD will touch the Circle.

PROPOSITION XVIII.

A THEOREM.

A Line drawn from the Centre of a Circle to the Point where a right Line touches it, is perpendicular to that Line.



IF the Line CD be drawn from the Centre C to the Point of Contact D, CD will be perpendicular to AB. For if it be not, draw the Line CB perpendicular to AB.

Demonstration.

Since the Line CB is suppos'd to be perpendicular, the Angle B will be a right Angle, and consequently CDB an Acute, (*by the 32. 1.*) Therefore the Line CB, oppos'd to the lesser Angle, will be less than CD, which is impossible; because CF, which is but part of CB, is equal to CD.

PROPOSITION XIX,

A THEOREM.

If a Line perpendicular to the Tangent, be drawn from the Point of Contact, it will pass through the Centre of the Circle.

LET the Line AB [see Fig. *preced.*] touch the Circle at the Point D, and the Line DC be perpendicular to AB. I say, That DC passes through the Centre. For if it did not, drawing a Line from the Centre to the Point D, it would be perpendicular to AB, (*by the preceding.*) and so there would be two Perpendiculars drawn to the same Point D of the same Line CB, which cannot be.

The USE.

‘ The Use of Lines *Tangents* is very common
 ‘ in *Trigonometry*; upon which account it is that
 ‘ I have made a Table, whereby to measure all
 ‘ sorts of Triangles, as well Spherical as Recti-
 ‘ lineal. In my *Opticks* likewise are divers
 ‘ Propositions founded upon *Tangents*; as when
 ‘ it is determin’d what part of a Globe is en-
 ‘ lighten’d. The *Phases* or Apparitions of the
 ‘ Moon are establish’d also upon the same Doc-
 ‘ trine; and that famous Problem of *Hipparchus*,
 ‘ by which he found the Distance of the Sun,
 ‘ by the Difference of the true and apparent
 ‘ *Quadratures*. In *Dialling*, the *Italian* and Ba-
 ‘ bylonian

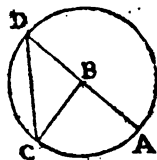
' *bylonian* Hours are frequently describ'd by
' Lines Tangents. Lastly, We take the Di-
' mensions of the Earth by a Line that touches
' its Superficies; and in the Art of *Navigation*,
' take a Tangent Line for our Horizon.

PROPOSITION XX.

A THEOREM.

The Angle at the Centre is double the Angle at the Circumference, which has the same Arch for its Base.

IF the Angle ABC , which is at the Centre, and the Angle ADC , at the Circumference, have the same Arch AC for their Base, the first will be double the second. This Proposition has three different Cases: The first of which is, when the Line ABD passes through the Centre B , the Line AB in one Triangle concurring with the Line BD of the other.



Demonstration.

The Angle ABC is an external Angle in respect of the Triangle BDC : Therefore (*by the 32. 1.*) it is equal to both the Angles D and C , which being equal, (*by the 5. 1.*) because their Sides BC and BD are equal, the Angle ABC is the double of either.

The

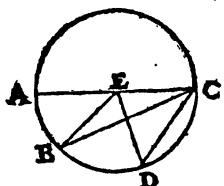


The Second Case is, When one Angle incloses the other, but none of the Lines that form them concur in one; as you see in the Figure annex'd. The Angle BED is at the Centre, and the Angle BAD at the Circumference. Draw the Line AIC through the Centre.

Demonstration.

The Angle BIC is double the Angle BAC , and CID is double the Angle CAD , (*by the preceding Case:*) Therefore the Angle BID is double the Angle BAD .

The third Case is, When it happens that neither one Angle incloses the other, nor do any of the Lines that form them, concur in one. Which Case is wholly omitted by my Author, but for the Readers Satisfaction is here supplied.



Let the Angle at the Centre be BED , and the Angle at the Circumference BCD , having the same Arch for their Base BD . I say, the Angle BED is double the Angle BCD . Draw the Line EC , and continue it to the Point A .

Demonstration.

The Angle AED is double the Angle ACD , (*by the 1. Case;*) and (*by the same*) the Angle AEB is double the Angle ACB : Therefore the Remainder of the one BED is double the Remainder of the other BCD .

The

The USE.

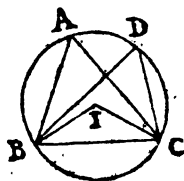
‘ That Problem, which is ordinarily propos’d,
 ‘ shewing how to describe an *Horizontal Dial*
 ‘ by one sole opening of the Compass, is built
 ‘ in part on this Proposition. And again, When
 ‘ we would determine the *Apogæum* of the Sun,
 ‘ or the Excentricity of his Circle, by three
 ‘ Observations, we suppose the Angle at the
 ‘ Centre to be double that at the Circumfe-
 ‘ rence. *Ptolemy* makes frequent Use of this
 ‘ Proposition, to determine both the Excentric
 ‘ Circle of the Sun, and the Epicycle of the
 ‘ Moon. The first Proposition of the Third
 ‘ Book of *Trigonometry* is grounded also upon
 ‘ this here.

PROPOSITION. XXI.

A THEOREM.

*The Angles that are in the same Segment of a Circle,
 or that have the same Arch for their Base, are
 equal.*

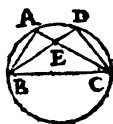
IF the Angles BAC and BDC
 are in the same Segment of
 a Circle, which is greater than
 a Semi-circle, they will be e-
 qual. Draw the Lines BI and
 CI.



Demon-

Demonstration.

The Angles A and D are each of them the half of the Angle BIC, (*by the preceding.*) therefore they are equal. They have likewise the same Arch BC for their Base.



Secondly, Let the Angles A and D be in the same Segment BAD, which is less than a Semicircle; they will nevertheless be equal.

Demonstration.

All the Angles of the Triangle ABE are equal to all the Angles of the Triangle DEC, (*by the 1. Coroll. of the 32. 1.*) but the Angles AEB and DEC are equal, (*by the 15. 1.*) Also the Angles ECD and ABE are equal, (*by the preceding Case,*) being in the same Segment ABCD, greater than a Semi-circle; therefore the Angles BAE and EDC are equal; which, the Angles at E being equal, and consequently (*by the Coroll. of the 15. 1.*) the Lines AE and EC, making but one right Line, as likewise DE and EB another; are the Angles A and D, in the same Segment ABCD, and having the Arch BC for their Base.

The USE.

‘ This Proposition is produc’d in *Opticks*, to prove, that the Line BC will appear of the same Greatness, when ’tis view’d from A, and D, because it is seen in both Cases under equal Angles.

‘ The same Proposition is us’d to describe large Circles without having their Centres:

‘ For

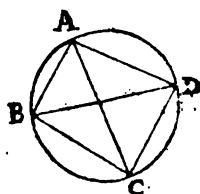
‘ For Example ; if we would make large Cop-
 ‘ per Basons of a Spherical Figure, such as we
 ‘ might work upon in Polishing Spectacles and
 ‘ Glasses to see at a great Distance. For hav-
 ‘ ing made in Iron an Angle BAC equal to that
 ‘ which is contain’d in the Segment ABC, and
 ‘ at the Points B and C strongly fasten’d two
 ‘ small Iron Pins, if the Triangle BAC be mov’d
 ‘ so, that the Side AB may always touch the
 ‘ Pin B, and the Side AC the Pin C, the Point
 ‘ A will describe an Arch of the Circle ABCD.
 ‘ This manner of describing a Circle may also
 ‘ be us’d in making great *Astrolabes*.

PROPOSITION XXII.

A THEOREM.

*Quadrilateral Figures, inscrib’d in a Circle, have
 their opposite Angles equal to two right Angles.*

LET a Quadrilateral Fi-
 gure, or a Figure of four
 Sides, be inscrib’d in a Circle,
 in such sort that all its An-
 gles may terminate at the
 Circumference of the Circle
 ABCD: I say, The opposite
 Angles BAD and BCD are equal to two right
 Angles. Draw the Diagonals AC, and BD.



Demon-

Demonstration:

All the Angles of the Triangle BAD are equal to two right Angles. Instead then of the Angle ABD put the Angle ACD, which is equal to it (*by the 21.*) being in the same Segment ABCD: And instead of the Angle ADB, put the Angle ACB, which is in the same Segment of the Circle BCDA: Therefore the Angle BAD, and the Angles ACD and ACB, that is to say, the whole Angle BCD, are equal to two right Angles.

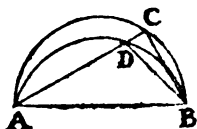
The USE.

‘ *Ptolemy* makes Use of this Proposition to
 ‘ frame the Table of Chords, or Lines subtend-
 ‘ ing Arches. I have also us’d the same in my
 ‘ third Book of *Trigonometry*, to prove, that
 ‘ the Sides of an Obtuse-Angle Triangle have
 ‘ the same Proportion among themselves as the
 ‘ Sines of the Opposite Angles.

PROPOSITION XXIII.

A THEOREM.

Two Similar Segments of a Circle, describ’d upon the same Line, are equal.



I Call those Similar Segments of a Circle, which contain equal Angles; and I say, that if such be describ’d upon the same Line AB, they will fall
 one

one upon the other, and not exceed each other in any Part. For if either did exceed the other, as do the Segments ADB, and ACB, they would not be Similar: To demonstrate which, draw the Lines ADC, BD and BC.

Demonstration.

The Angle ADB is an external Angle, in respect of the Triangle DBC: Therefore (by the 16. 1) it is greater than the Angle ACB; and by consequence the Segments ADB and ACB contain unequal Angles, which, I say, is to be *Dissimilar*.

PROPOSITION XXIV.

A THEOREM.

Two Similar Segments of a Circle describ'd upon equal Lines, are equal.

IF the Segments of the Circles, AEB and CFD, be similar, and the Lines AB and CD equal, the Segments also will be equal.

Demonstration.

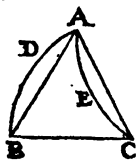
Suppose the Line CD to be plac'd upon the Line AB, being suppos'd to be equal, they will not exceed each other; and then the Segments AEB and CFD will be describ'd upon the same Line, and therefore will be equal, (by the preceding.)



The

The USE.

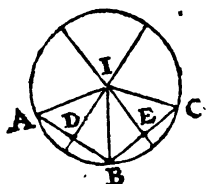
‘ Crooked Figures are frequently reduc’d to
 ‘ Rectilineals by this Proposition. As for Ex-
 ‘ ample; If two Similar Segments
 ‘ of a Circle AEC and ADB, be
 ‘ describ’d upon AB and AC, the
 ‘ equal Sides of the Triangle ABC:
 ‘ ’tis evident, that, transposing the
 ‘ Segment AEC unto ADB, the
 ‘ Triangle ABC is equal to the Figure
 ‘ ADBCEA.



PROPOSITION XXV.

A PROBLEM.

To complet a Circle of which we have but a Part.



HAVING the Arch ABC given, to complet the Circle we must find its Centre; to which end, draw the Lines AB and BC, which having divided in the Middles at the Points E and D, draw their two Perpendiculars EI and DI; which will meet at the Point I, the Centre of the Circle.

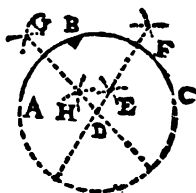
Demonstration.

The Centre is in the Line DI, (by the Coroll. of the 1.) it is also in EI, (by the same) therefore it must be at the Point I.

The

The USE.

' This Proposition occurs very frequently ;
 ' but sometimes it is express'd in other Terms :
 ' As to inscribe a Triangle in a Circle ; or to
 ' describe a Circle through
 ' three Points given, provi-
 ' ded they be not plac'd in a
 ' right Line. Let the Points
 ' propos'd be A, B, and C, and
 ' placing the Foot of the Com-
 ' pass at the Point C, de-
 ' scribe two Arches F and E, at any Distance
 ' whatsoever. Then remove the Foot of the
 ' Compass to the Point B, and at the same Dis-
 ' tance describe two other Arches cutting the
 ' former in E and F ; also from the Point B, as
 ' the Centre, describe at any Distance the
 ' Arches G and H, and at the same Distance from
 ' the Centre A two other Arches cutting them
 ' in G and H. Which done, draw the Lines
 ' through F and E, G and H, which shall cut
 ' each other at the Point D, the Centre of the
 ' Circle. The Demonstration is obvious enough ;
 ' for if you had drawn the Lines AB and BC,
 ' you had, by this Operation, divided them e-
 ' qually and perpendicularly. This Proposi-
 ' tion is exceeding necessary to describe *Astro-*
 ' *labes*, and compleat Circles, of which we have
 ' but three Points. That Proposition in *Astro-*
 ' *nomy*, which teaches how to find the *Apogæum*,
 ' and Excentricity of the Circle of the Sun,
 ' virtually contains this. And I also have
 L made



made frequent Use of it in my Treatise concerning the cutting of Stones.

PROPOSITION XXVI.

A THEOREM.

Equal Angles, whether at the Centres, or the Circumferences of equal Circles, have equal Arches for their Bases.



IF the Angles D and I, at the Centres of equal Circles ABC, and EFG, be equal; the Arches BC and FG will be equal. For if the Arch BC was greater or less than the Arch FG, since the Angles are measur'd by Arches, the Angle D would be greater or less than the Angle I.

But if the equal Angles be suppos'd to be at the Circumferences of equal Circles, as A and E; the Angles which they inclose at the Centres, as D and I, being their Doubles, will be likewise equal, and consequently require equal Arches for their Bases, BC, and FG; which Arches are likewise the Measures of the Angles A and E.

PRO-

PROPOSITION XXVII.

A THEOREM.

Angles, whether at the Centres or Circumferences of equal Circles, having equal Arches for their Bases, are also equal.

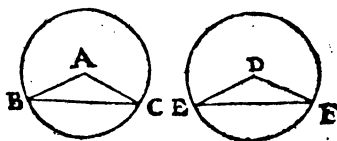
IF the Angles D and I (*Fig. preced.*) at the Centres of equal Circles have equal Arches BC and FG for their Bases, they will be equal, because their Measures BC and FG are equal. And if the Angles A and E, at the Circumferences of equal Circles, have equal Arches BC and FG for their Bases, since the Angles they inclose at the Centres will be equal, they also that are the Halves of those Angles (*by the 20*) will be equal.

PROPOSITION XXVIII.

A THEOREM.

Equal Lines, within equal Circles, answer to equal Arches.

IF the equal Lines BC and EF be applied to equal Circles, ABC, and DEF, they



L 2

will

will be the Chords of equal Arches, BC , and EF . Draw the Lines AB , AD , DE , DF .

Demonstration.

In the Triangles ABC and DEF , the Sides AB and AC , DE and DF , are equal, being the Semi-diameters of equal Circles; and their Bases BC and EF are suppos'd equal; therefore (by the 8. 1.) the Angles A and D will be equal; and (by the 26.) the Arches BC and EF will be also equal.

PROPOSITION XXIX.

A THEOREM.

The Lines that subtend equal Arches of equal Circles are equal.

IF the Lines BC and EF (see Fig. preced. Prop.) subtend (or are the Chords of) equal Arches BC and EF in equal Circles, they will be equal.

Demonstration.

The Arches BC and EF are equal, and Parts of equal Circles; therefore (by the 27.) the Angles A and D will be equal: Therefore in the Triangles ABC , DEF , the Sides AB , AC , DE , and DF , being equal, as also the Angles A and D ; the Bases BC , EF , will be equal, (by the 4. 1.)

The USE.

‘ *Theodosius*, by the 28 and 29, demonstrates,
‘ that the Arches of the Circles of the *Italian*
‘ and *Babylonian* Hours, contain'd between
‘ two

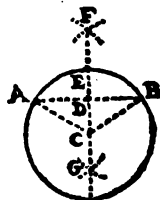
two Parallels, are equal. We have also demonstrated after the same manner, that the Arches of the Circles of the *Astronomical* Hours, contain'd between two Lines parallel to the *Equator*, are likewise equal. These Propositions are almost of continual Use in *Spherical Trigonometry*, and also in *Dialling*.

PROPOSITION XXX.

A PROBLEM.

To divide an Arch of a Circle into two equal Parts.

Suppose the Arch AEB was to be divided into two equal Parts; place the Foot of the Compass at the Point A, and describe two Arches F and G; then removing it to the Point B, at the same Distance describe other two Arches, cutting the former in F and G; the Line GF will cut the Arch AB equally at the Point E. Draw the Line AB.



Demonstration.

By this Operation we have divided the Line AB into two equal Parts. For suppose there were drawn the Lines AF, BF; AG, and BG; (which I have not done, lest the Figure should appear confus'd,) the Triangles FGA and FGB would have all their Sides equal; there-

fore (*by the 8. 1.*) the Angles $A F D$ and $B F D$ would be equal. Again, The Triangles $D F A$ and $D F B$ have the Side DF common, the Sides AF and BF equal, and the Angles DFA and DEB equal: Therefore (*by the 4. 1.*) the Bases AD and BD are equal; and also the Angles ADF and BDF . We have therefore divided the Line AB equally, and perpendicularly at the Point D : Therefore (*by the 1.*) the Centre of the Circle is in the Line FG . Suppose it then to be the Point C , and draw the Lines CA and CB ; all the Sides of the Triangles ACD and BCD are equal; therefore (*by the 8. 1.*) the Angles ACD and BCD are equal, and (*by the 26.*) the Arches AE and EB .

The U S E.

‘ Having frequent Occasion to divide an Arch
 ‘ into two equal Parts, the Exercise of this Pro-
 ‘ position is very common. ’Tis thus that we
 ‘ divide the Mariners Compass into 32 Winds:
 ‘ For having drawn two Diameters cutting each
 ‘ other at right Angles, we divide the Circle in
 ‘ four Parts; and subdividing each Quarter in
 ‘ the Middle, we have eight Parts; and again,
 ‘ subdividing those twice, we make 32. We
 ‘ have also Occasion for the same Operation in
 ‘ the dividing a Semi-circle into 180 Degrees;
 ‘ and, because to compleat that Division we are
 ‘ oblig’d to divide an Arch into three equal Parts;
 ‘ all Geometricians have sought after a Method
 ‘ of doing that Geometrically, but have not yet
 ‘ been so happy as to find one.

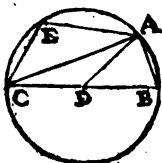
P R O.

PROPOSITION XXXI.

A THEOREM.

The Angle in a Semi-circle is a right Angle; that which is in a Segment greater than a Semi-circle is an Acute; and that which is in a lesser Segment is an Obtuse.

IF the Angle BAC be in a Semi-circle, I will prove that it is a right Angle. Draw the Line DA.



Demonstration.

The Angle ADB being an external Angle in regard of the Triangle DAC, is equal to both the Internals DAC and DCA, (*by the 32. 1.*) and those being equal (*by the 5. 1.*) because the Sides DA and DC are equal, it will be double the Angle DAC. In like manner the Angle ADC is double the Angle DAB: Therefore the two Angles ADB and ADC, which are equal to two right Angles, are double the whole Angle BAC, and consequently the Angle BAC is a right Angle.

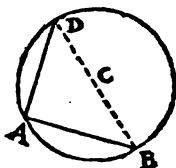
Secondly, The Angle AEC, which is in the Segment AFC less than a Semi-circle, is an Obtuse Angle. For in the Quadrilateral Figure ABCE, the two opposite Angles E and B are equal to two right Angles, (*by the 22.*) but the Angle B is an Acute; therefore the Angle E will be an Obtuse.

L 4

Thirdly,

Thirdly, The Angle B, which is in the Segment ABC greater than a Semi-circle, is an Acute; because in the Triangle ABC, the Angle BAC is a right Angle.

The USE.



‘ *Mechanicks* make Use of this Proposition to try if their Squares be just; for having describ’d a Semi-circle BAD, they lay down the Point A of their Square BAD upon the Circumference, and one of its Sides AB upon the Point of the Diameter B; and then the other Branch AD ought to pass precisely through the Point D, which is the other Extreme of the Diameter.

‘ *Ptolemy* uses this Proposition to compose his Table of Chords or Subtendents, of which he has occasion in his *Trigonometry*.

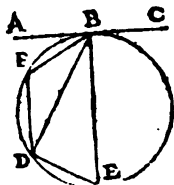
‘ There is also a Method of raising a Perpendicular at the end of a Line, grounded upon this Proposition. For Example; To raise a Perpendicular at the Point A of the Line AB, I place the Foot of the Compass upon the Point C taken any-where, and describe a Circle through the Point A, cutting the Line AB at the Point B. Then I draw the Line BCD; and so ’tis evident, that the Line AD is in a Semi-circle.

PROPOSITION XXXII.

A THEOREM.

A Line cutting a Circle at the Point of Contact, makes, with the Tangent, Angles equal to those in the alternate Segments.

LET the Line BD cut the Circle at the Point B, which is that where the Line AB touches it. I say, The Angle CBD, made by the Line BD, and the Tangent ABC, is equal to the Angle F in the alternate Segment BFD; and that the Angle ABD is equal to the Angle E in the Segment BDE.



First, If the Line pass through the Centre, as the Line BE, it will make, with the Tangent, two right Angles, (*by the 18.*) and the Angles of the Semi-circles would be also right Angles, (*by the preceding;*) therefore in this Case the Proposition would be true. But if the Line do not pass through the Centre, as BD; draw the Line BE through the Centre, and join the Line DE.

Demonstration.

The Line BE makes with the Tangent two right Angles; and all the Angles of the Triangle BDE are equal to two right Angles, (*by the 32. 1.*) therefore taking away the right Angles CBE, and D which is in the Semi-circle, and likewise the Angle EBD which is common to both, there will remain the Angle ABD equal to the Angle E.

Again,

Again, The Angle CBD is equal to the Angle F; because in the Quadrilateral Figure BFDE, which is inscrib'd in a Circle, the opposite Angles E and F are equal to two right Angles, (*by the 22.*) but the Angles ABD and CBD are also equal to two right Angles, (*by the 13. 1.*) and the Angles ABD and E are equal, as I have now demonstrated; therefore the Angles CBD and F are equal.

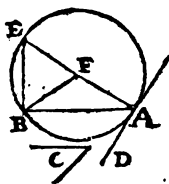
The USE,

' This Proposition is necessary to prove that which follows.

PROPOSITION. XXXVII.

A PROBLEM.

Upon a Line given, to describe a Segment of a Circle capable of an Angle given.



LET it be propos'd to describe a Segment of a Circle upon the Line AB capable of the Angle C. Make the Angle BAD equal to the Angle C, and draw AE perpendicular to AD; make also the Angle ABF equal to the Angle BAF: And in fine, from the Point F, where BF and AE concur, at the distance BF, or FA, describe a Circle. The Segment BEA is capable of an Angle equal to the Angle C.

Demon.

Demonstration.

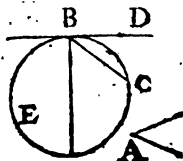
The Angles BAF and ABF being equal, the Lines FA and FB are equal, (*by the 6. 1.*) and the Circle, which is describ'd from the Centre F, by A, passes by B: Now the Angle DAE being a right Angle, the Line DA touches the Circle in A, (*by the 16.*) therefore the Angle contain'd in the Segment BEA, as the Angle E, is equal to the Angle DAB; that is, the Angle C, (*by the preceding.*) But if the Angle given be an Obtuse, we must take an Acute its Complement to 180 Degrees,

PROPOSITION XXXIV.

A PROBLEM.

A Circle being given, to cut a Segment from it capable of a certain Angle.

TO cut a Segment of the Circle BCE capable of the Angle A, draw (*by the 17.*) the Tangent BD, and make the Angle DBC equal to the Angle A. 'Tis evident (*by the 32.*) that the Segment BEC is capable of an Angle equal to DBC, and consequently to the Angle A.



The USE.

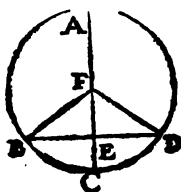
‘ I have made Use of this Proposition to find
 ‘ Geometrically the Excentricity of the Annual
 ‘ Circle of the Sun, and his *Apogæum*, having
 ‘ three Observations given. ’Tis used likewise
 ‘ in *Opticks*, to find a Point where two unequal
 ‘ Lines propos’d, may appear equal, or under
 ‘ equal Angles, by making upon each Line
 ‘ Segments which will contain equal Angles.

PROPOSITION XXXV.

A THEOREM.

If two Lines cut each other within a Circle, the Rectangle contain’d under the Parts of one, is equal to the Rectangle contain’d under the Parts of the other.

First, If the two Lines cut each other in the Centre, they will be both equal, and both equally divided; so that in this Case it is evident, the Rectangle contain’d under the Parts of one, will be equal to the Rectangle contain’d under the Parts of the other.



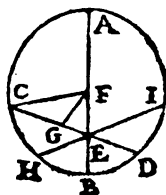
Secondly, If one of the Lines pass through the Centre F, as AC, and divide the Line BD into two equal Parts at the Point E: I say, The Rectangle contain’d under AE and EC is equal to the Rectangle contain’d under

under BE and ED, that is to say, to the Square of BE. The Line AC is perpendicular to BD, (by the 3.)

Demonstration.

Since the Line AC is divided equally in F, and unequally in E, the Rectangle contain'd under AE, and EC, with the Square of FE is equal to the Square of FC, or FB, (by the 5. 2.) Now the Angle E being a right Angle, the Square of FB is equal to the Squares of BE and FE; therefore the Rectangle under AE, EC, with the Square of E, is equal to the Squares BE and FE; and taking away the Square of EF, there remains the Rectangle under AE, EC, equal to the Square of BE.

Thirdly, Let the Line pass through the Centre F, and divide the Line CD into unequal Parts at the Point E: Draw FG perpendicular to CD, and (by the 3.) the Lines CG and GD will be equal.



Demonstration.

Since the Line AB is divided equally in F, and unequally in E, the Rectangle contain'd under AE, EB, with the Square of EF, is equal to the Square of FB, or FC, (by the 5. 2.) Instead of the Square of EF, put the Squares of FG and GE, which are equal to it, (by the 47. 1.)

In like manner the Line CD being divided equally in G, and unequally in E; the Rectangle

angle under CE, ED, with the Square of GE, is equal to the Square of GC. Add the Square of GF; the Rectangle under CE, ED, with the Squares of GE and GF, will be equal to the Squares of CG and GF, that is to say, (*by the 47. 1.*) to the Square of FC. Therefore the Rectangle under AE, EB, with the Squares of EG and GF, is equal to the Rectangle under CE, ED, with the same Squares: And consequently taking away the same Squares from both, the Rectangle AE, EB, will be equal to the Rectangle CE, ED.

Fourthly, If the Lines CD and HI, cut each other in E, neither of the two passing through the Centre: I say, The Rectangle CE, ED is equal to the Rectangle HE, EI. For drawing the Line AFB, it is plain the Rectangles CE, ED, and HE, EI, are both equal to the Rectangle AE, EB, (*by the preceding Case*;) therefore they are equal betwixt themselves.

The USE.

‘ We are taught by this Proposition a Method of finding a fourth Proportional to three Lines given, or a third Proportional to two.

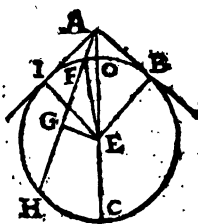
PRO-

PROPOSITION XXXVI.

A THEOREM.

If from a Point taken without the Circle a Line be drawn to touch, and another to cut the Circle; the Square of the Tangent will be equal to the Rectangle contain'd under the whole Secant, and the external Line.

Suppose the Line AB to be drawn from the Point A, taken without the Circle, to touch the Circle in B; and the Line AC, or AH cutting it: The Square of AB will be equal to the Rectangle contain'd under AC and AO, as also to the Rectangle contain'd under AH and AF. If the Secant pass through the Centre, as AC, draw the Line EB.



Demonstration.

Since the Line OC is divided in the middle at the Point E, and the Line AO added to it; the Rectangle contain'd under AO and AC, with the Square of OE, or EB, will be equal to the Square of AE, (*by the 6. 2.*) Now the Line AB is suppos'd to touch the Circle at the Point B: Therefore (*by the 18.*) the Angle B is a right Angle, and (*by the 47. 1.*) the Square of AE is equal to the Squares of EB and

and AB; therefore the Rectangle under AC and AO, with the Square of EB, is equal to the Squares of EB and AB: And taking away the Square of EB from both, the Rectangle under AC, AO will be equal to the Square of AB.

Secondly, Suppose the Secant AH not to pass through the Centre; and draw the Line EG perpendicular to FH, which will divide in the Middle the Line FH at the Point G; draw also the Line EF.

Demonstration.

The Line FH being divided equally at the Point G, and the Line AF being added to it; the Rectangle contain'd under AH, AF, with the Square of FG, will be equal to the Square of AG. Add to both the Square of EG; the Rectangle under AH, AF, with the Squares of FG and GE, that is, (*by the 47. 1.*) the Square of FE, or EB, will be equal to the Squares of AG and GE, that is, (*by the 47. 1.*) the Square of AE. Further, The Square of AE (*by the same*) is equal to the Squares of EB and AB; Therefore the Rectangle contain'd under AH, AF, with the Square of BE, is equal to the Squares of BE and AB: and taking away the Square of BE from both, the Rectangle contain'd under AH, AF will be equal to the Square of AB.

Coroll. 1. If you draw divers Secants from the same Point, as AC and AH, the Rectangles under AC and AO, AH and AF, will be equal

equal betwixt themselves, since they are both equal to the Square of AB.

Coroll. 2. If you draw two Tangents from the same Point, as AB, AI, they will be equal; because the Squares will be equal to the same Rectangle under AC and AO, and consequently betwixt themselves; as also the Lines AB, AI.

PROPOSITION XXXVII.

A THEOREM.

If the Rectangle contain'd under the Secant and the External Line be equal to the Square of a Line that falls upon the Circle, that Line will touch the Circle.

Suppose the Secant to be AC, or AH, and the Rectangle AC, AO; or AH, AF, (*see Fig. preced.*) to be equal to the Square of the Line AB; the Line AB will touch the Circle. Draw the Tangent AI, (*by the 17.*) and the Line IE.

Demonstration.

Since the Line AI touches the Circle, the Rectangle AC, AO; or AH, AF, will be equal to the Square of AI. But the Square of AB is suppos'd to be equal to either of those Rectangles; therefore the Squares of AI and AB are equal, and consequently the Lines AI and

M

AB.

AB. Therefore the Triangles ABH and AIE, having all Sides equal, will be equiangular, (*by the 8. 1.*) and because the Angle AIE is a right Angle (*by the 18.*) the Line AI being a Tangent, the Angle ABE will be a right Angle, and the Line AB a Tangent, (*by the 16.*)

The USE.

‘ *Maurylochus* makes use of this Proposition to
 ‘ find the Diameter of the Earth. For observ-
 ‘ ing from the Top of a Mountain OA, the Su-
 ‘ perficies of the Earth by the Line BA, he takes
 ‘ notice of the Angle OAB, made by the Line
 ‘ AB and a Perpendicular AC: And by *Trigo-*
 ‘ *nometry* calculates the Length of the Line
 ‘ AB. Then multiplying AB by AB to have
 ‘ its Square, he divides the Product by AO the
 ‘ Height of the Mountain, which gives the
 ‘ Quotient AC, the Diameter of the Earth,
 ‘ with the Height of the Mountain; from
 ‘ which having subducted AO, there will re-
 ‘ main OC the Diameter of the Earth. This
 ‘ Proposition serves also to prove the fifth of the
 ‘ third Book of *Trigonometry*.

THE
FOURTH BOOK
OF THE
Elements of *EUCLID*.

THIS *Fourth Book* is exceeding useful in *Trigonometry*. For by inscribing Polygons in a Circle, we learn the Methods of Composing the Tables of Subtendents, Tangents, and Secants; a Practice most necessary for taking all sorts of Dimensions.

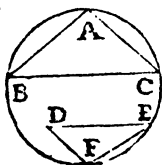
Again, By inscribing Polygons in a Circle, we find the divers Aspects of the Stars, which also take their Names from those Polygons.

Thirdly, The same Operations give us the *Quadrature* of the Circle, as exact as is needful. And by them we also demonstrate, that Circles are in the duplicate Proportion of that of their Diameters.

Fourthly, *Military Architecture* does frequently

requently require to inscribe Polygons in a Circle, to compose the Designs and Platforms of regular Fortifications.

D E F I N I T I O N S.



A Rectilineal Figure is inscrib'd in a Circle, or a Circle is describ'd about it, when all its Angles are in the Circumference of the same Circle.

As the Triangle ABC is describ'd in a Circle, and the Circle is inscrib'd about the Triangle; because its Angles A, B, and C, do all terminate at the Circumference. The Triangle DEF is not inscrib'd in the Circle, because the Angle D does not terminate at the Circumference of the Circle.

2. A Rectilineal Figure is inscrib'd about a Circle, and a Circle describ'd within that Figure, when all the Sides of the

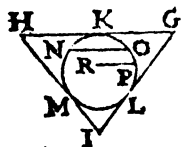


Figure touch the Circumference of the Circle. As the Triangle, GHI is describ'd about the Circle KLM, because its Sides touch the Cir-

cumference of the Circle in K, L, and M.

3. A Line is apply'd to, or inscrib'd in a Circle,

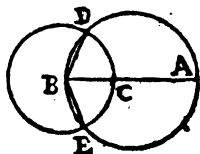
Circle, when its two Extreame touch the Circumference of the Circle. ' As the Line NO.
' But the Line RP is not inscrib'd in the Circle.

PROPOSITION I.

A PROBLEM.

To inscribe in a Circle a Line, that does not exceed its Diameter.

LET a Line be propos'd to be inscrib'd in the Circle AE $\overline{B}$ D, not exceeding its Diameter. Take the length of the Line propos'd upon the Diameter; for Example, Let it be BC. Place the Foot of the Compass upon the Point B, and describe a Circle at the Distance of BC, which may cut the Circle AE $\overline{B}$ D in D and E. Then draw the Line BD, or BE. 'Tis evident they are equal to BC, (*by the Definition of a Circle.*)



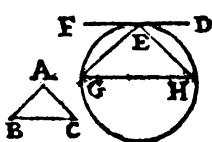
The USE.

' This Proposition is necessary for the Performance of what is requir'd in the following.

PROPOSITION II.

A PROBLEM.

To inscribe in a Circle a Triangle equiangular to another Triangle.



LET the Circle be EGH, in which a Triangle is to be inscrib'd, equiangular to the Triangle ABC. Draw the Tangent FED, (by the 17. 3.) and at the Point of Contact E make the Angle DEH equal to the Angle B, and the Angle FEG equal to the Angle C, (by the 23. 1.) and draw the Line GH; the Triangle EGH will be equiangular to the Triangle ABC.

Demonstration.

The Angle DEH is equal to the Angle EGH of the alternate Segment, (by the 32. 3.) But the Angle DEH is equal to the Angle B, and consequently the Angles B and G are equal. By the same reason the Angles C and H are also equal, and (by Coroll. 2. of the 32. 1.) the Angles A and GEH will be equal. Therefore the Triangles EGH and ABC are equiangular.

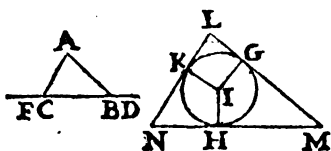
P R O-

PROPOSITION III.

A PROBLEM.

To describe a Triangle about a Circle, Equiangular to another Triangle.

IF you would describe a Triangle Equiangular to the Triangle A B C about the Circle GKH. Con-



tinue one of the Sides of the Triangle given BC to D and F, and make the Angle GIH equal to the Angle ABD, and, HIK equal to the Angle ACF: Then draw the Tangents LGM, LKN, and NHM through the Points G, K, and H. These Tangents will concur; because the Angles IKL and IGL being right Angles, if you should draw a Line KG, the Angles KGL and GKL would be less than two right Angles: Therefore the Lines GL and KL must concur, (by the 11. Axiom.)

Demonstration.

All the Angles of the Quadrilateral $GIHM$ are equal to four right Angles, because it may be divided into two Triangles. The Angles IGM and IHM , which are made by

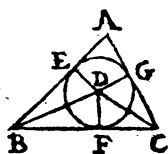
M 4 the

the Tangents are right Angles; therefore the Angles M and I are equal to two right Angles, as are also the Angles ABC, and ABD. But the Angle GIH is equal to the Angle ABD, therefore the Angle M will be equal to the Angle ABC. By the same reason the Angles N and ACB are equal, and therefore the Triangles LMN and ABC are equiangular.

PROPOSITION IV.

A PROBLEM,

To inscribe a Circle in a Triangle,



IF you would inscribe a Circle in the Triangle ABC, divide the Angles ABC and ACB into two equal Parts, (*by the 9. 1.*) drawing the Lines BD and CD, which will concur at the Point D. This done, from the Point D draw the Perpendiculars DE, DF, and DG, which will be equal; so that a Circle describ'd from the Centre D, at the Distance DE, will pass through F and G.

Demonstration.

The Triangles DEB and DFB have the Angles DEB and DFB equal, being both right Angles: The Angles DBE and DBF are also equal,

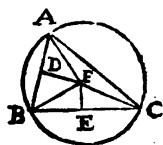
equal, the Angle A E C having been divided into two equal Parts; and the Side D B is common: therefore (*by the 26. 1.*) the Triangles will be equal in all Respects, and the Sides DE and DF will be equal. After the same manner might I demonstrate the Sides DF and DG to be equal. 'Tis possible therefore to describe a Circle which shall pass through the Points E, F, and G; and because the Angles E, F, and G are right Angles, the Sides AB, AC, and BC will touch the Circle, which by consequence is inscrib'd in the Triangle.

PROPOSITION V.

A PROBLEM.

To describe a Circle about a Triangle.

IF you would describe a Circle about the Triangle ABC, divide the Sides AB and BC into two equal Parts, at the Points D and E, drawing the Perpendiculars DF and EF, which will concur at the Point F. Which done, if you describe a Circle from the Center F at the Distance FB, it will pass through A and C; that is to say, the Lines FA, FB, and FC are equal.



Demonstration.

The Triangles A D F and B D F have the Side

Side DF common, and the Sides AD and DB equal, the Side AB having been divided equally in D; and the Angles at D are equal, being right Angles. Therefore (*by the 4. 1.*) the Bases AF and BF are equal; as also the Bases BF and CF.

The USE.

‘ I have frequent Occasion to inscribe a Triangle in a Circle; as, for Instance, in the first Proposition of my third Book of *Trigonometry*. This Performance also is necessary for measuring the *Area* of a Triangle; and upon many other Occasions.

PROPOSITION. VI.

A PROBLEM.

To inscribe a Square in a Circle.



TO inscribe a Square in the Circle ACBD, draw the Diameter AB, and perpendicular to it the Line DC passing through the Center E; then draw the Lines AC, CB, BD, DA, and you will have inscrib'd in the Circle the Square ACBD.

Demonstration.

The Triangles AEC and CEB have their Sides equal, and the Angles AEC and CEB equal,

qual, being both right Angles : therefore their Bases AC and CB are equal, (*by the 4. 1.*) Further, because the Sides AE and EC are equal, the Angles EAC and ECA will be equal; and the Angle E being a right Angle they will be half-right Angles, (*by the 32. 1.*) therefore the Angle ECB is half a right Angle, and consequently the Angle ACB will be a right Angle. And the same Reason holds for all the rest: therefore the Figure ACBD is a Square.

PROPOSITION VII.

A PROBLEM.

To describe a Square about a Circle.

HAVING drawn the two Diameters AB and CD, which cut each other perpendicularly at the Center E, draw the Tangents FG, GH, HI, and IF, by the Points A, D, B, C, and you will have describ'd the Square FGHI about the Circle ACBD.



Demonstration.

The Angles E and A are right Angles, therefore (*by the 27. 1.*) the Lines FG and CD are Parallels. After the same manner I may prove, that CD and HI, FI and AB, AB and GH, are Parallels. Therefore the Figure

Figure FCDG is a Parallelogram, and (*by the 34. 1.*) the Lines FG and CD are equal, as also the Lines CD and IH, FI and AB, AB and GH; and consequently the Sides of the Figure FG and HI are equal. Further, since the Lines FG and CD are Parallels, and the Angle CDG is a right Angle, the Angle G will also be a right Angle (*by the 29. 1.*) After the same manner I may demonstrate the Angles F, H, and I, to be right Angles. Therefore the Figure FGHI is a Square, whose Sides touch the Circle.

PROPOSITION VIII.

A PROBLEM.

To inscribe a Circle in a Square.

IF you would inscribe a Circle in the Square FGHI, [*see Fig. preced.*] divide the Sides FG, GH, HI, IF, in the Middle at the Points A, D, B, C, and draw the Lines AB and CD; which may cut each other at the Point E. I demonstrate that the Lines EA, ED, EB, and EC are equal, and the Angles A, D, B, C, right Angles: and that therefore you may describe a Circle from the Center E, which will pass through A, D, B, and C, and touch the Sides of the Square FGHI.

Demon-

Demonstration.

Since the Lines AB and GH do conjoin the Lines AG and BH, which are parallel and equal, they also will be parallel and equal: therefore the Figure AGDE is a Parallelogram; and the Lines AE and GD, AG and ED are equal: and AG and GD being equal, AE and ED will be equal also. 'Tis after the same manner that the Lines AE, EC, EB, are prov'd equal. Further, AG and CD being parallel, and the Angle G a right Angle, the Angle D will be so likewise. Therefore the Circle ADBC may be describ'd from the Center E, which will pass through the Points A, D, B, C, and touch the Sides of the Square.

PROPOSITION IX.

A PROBLEM.

To describe a Circle about a Square.

TO describe a Circle about the Square ACBD, [see Fig. in Prop. 6.] draw the Diagonals AB and CD, which will cut each other at the Point E; the Point E will be the Center of the Circle, which will pass through the Points A, C, B, D. It ought therefore to be demonstrated, that the Lines AE, EB, CE, and ED are equal.

Demon-

Demonstration.

The Sides AC and CB are equal, and the Angle C is a right Angle; therefore the Angles BAC and ABC are equal, (*by the 5. 1.*) and half-right Angles, (*by the 32. 1.*)

After the same manner I demonstrate, that the Angles ACD, ADC, BDC, and BCD, are half-right Angles. Therefore the Triangle AEC having the Angles EAC and ECA half-right Angles, and consequently equal, will have also (*by the 6. 1.*) its Sides AE and EC equal. The same may be prov'd of the Lines EC and EB, EB, and ED, that they likewise are equal.

The U S E.

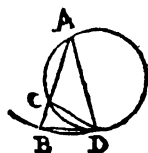
‘ We show in the 12th Book, that Polygons inscrib'd in a Circle, degenerate into Circles, and as these Polygons are always in the duplicate proportion of that of their Diameters, so likewise are Circles. In practical Geometry, we have frequent Occasion to inscribe a Square and other Polygons in a Circle, or to describe them about it, to reduce a Circle to a Square.

PROPOSITION X.

A PROBLEM.

To describe an Iſoſceles (or equicrural) Triangle having its Angles at the Baſe, each of them double to the third Angle.

TO deſcribe an Iſoſceles ABD, having each of its Angles ABD and ADB, double to the Angle A; divide the Line AB (by the 11. 2.) ſo that the Square of AC may be equal to the Rectangle under AB and BC; and from the Center A at the Diſtance AB deſcribe the Circle BD, in which inſcribe BD equal to AC; and drawing the Line DC, deſcribe a Circle about the Triangle ACD, (by the 5.)



Demonſtration.

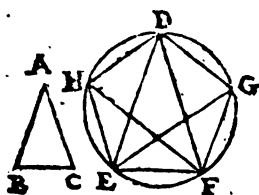
Since the Square of AC or BD is equal to the Rectangle contain'd under AB and BC, the Line BD will touch the Circle ACD at the Point D, (by the 37. 3.) therefore the Angle BDC will be equal to the Angle A, being in the alternate Segment CAD, (by the 32. 3.) Now the Angle BCD, being an external Angle in reſpect of the Triangle ACD, is equal to the Angles A and CDA; therefore the Angle BCD

BCD is equal to the Angle BDA. Further, the Angle ADB is equal to the Angle ABD, (*by the 5. 1.*) therefore DCB and DBC are equal, and (*by the 6. 1.*) the Sides BD and DC will be equal: and since BD is equal; and so likewise the Angles A and CDA. Therefore the Angle ADB is double the Angle A.

PROPOSITION XI.

A PROBLEM.

To inscribe a regular Pentagon in a Circle.



TO inscribe a regular Pentagon in a Circle; describe (*by the 10.*) an *Isoceles* ABC having each of its Angles ABC, ACB, at the Base, double to the Angle A. Inscribe in the Circle the Triangle DEF equiangular to ABC: then divide the Angles DEF and DFE into two equal Parts, drawing the Lines EG and FH. Lastly, join the Lines DH, DG, GF, EH, and you will have describ'd a regular Pentagon; that is to say, a Pentagon having equal Sides and equal Angles.

Demon-

Demonstration.

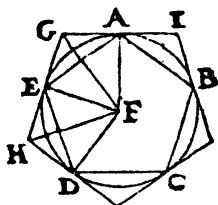
The Angles DEG, GEF, DFH, and HFE, being the Halves of the Angles DEF, and DFE, each of which is double to the Angle EDF, are equal to the Angle EDF; and consequently the five Arches, which are their Bases, are equal, (*by the 26. 3.*) and the Lines DH, HE, EF, FG, and GD, are equal, (*by the 29. 3.*) Secondly, the Angles DGF, GFE, and so of the rest, having each three of those equal Arches for its Base, will be also equal, (*by the 27. 3.*) Therefore the Sides and Angles of the Pentagon DHEFG are equal.

PROPOSITION XII.

A PROBLEM.

To describe a Pentagon about a Circle.

INscribe a regular Pentagon ABCDE in the Circle, (*by the 11.*) and having drawn Tangents through the Points A, B, C, D, E, (*by the 17. 2.*) you will have describ'd a regular Pentagon about the Circle. Draw the Lines FA, FG, FE, FH, FD.



Demonstration.

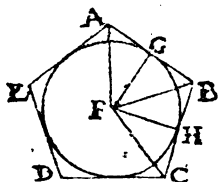
The Tangents GE and GA are equal, (*by Coroll. 2. of the 36. 3.*) as also EH and HD: the
N
Lines

Lines FA and FE are also equal (*by the Definit. of a Circle*;) therefore (*by the 8. 1.*) the Triangles FGA and FGE are equal in all respects; and the Angles AFG and EFG are equal, as also the Angles EFH and DFH. And because (*by the 27. 3.*) the Angles EFA and EFD are equal, their Halves EFH and EFG will be equal; and (*by the 26. 1.*) the Triangles EFH and EFG will be equal in all respects, and the Sides EG and EH will be also equal. After the same manner I can demonstrate all the Sides to be divided into two equal Parts; and consequently, since the Lines GE and GA are equal, GH and GI will be also equal. Further, the Angles G and H being double to the Angles FGE and FHE, are also equal. Therefore we have describ'd a regular Pentagon about the Circle,

PROPOSITION XIII.

A PROBLEM.

To inscribe a Circle in a regular Pentagon.



TO inscribe a Circle in the regular Pentagon ABCDE, divide the Angles A and B into two equal Parts by the Lines AF and BF, which concur at the Point F. Then drawing the Line FG perpendicular to AB, describe a Circle from the Center F at the Distance FG. I say it will touch all the other Sides, that

that is to say, having drawn FH perpendicular to BC, FH and FG will be equal.

Demonstration.

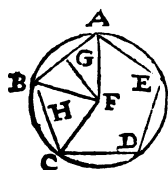
Since the equal Angles A and B were divided into two equal Parts, their Halves GAF and GBF will be equal: and since the Angles at G are right Angles, the Triangles AFG and BFG will be equal in all respects (*by the 26. 1.*) therefore the Lines AG and GB are equal.

Further, I may prove the Lines BG and BH, as also the Lines FG, and FH, to be equal; and the Sides AB and BC of a regular Pentagon being equal, the Lines BH and HC will be equal; and consequently the Angles at the Point H being also right Angles, and equal, the Triangle BFH and HFC will be equal, in all respects, and the Angles FBH and FCH will be equal. And since the Angles B and C, are equal, the Angle FCH will be half the Angle C. So passing from one to the other, I will demonstrate, that all the Perpendiculars FG and FH, and the rest, are equal.

PROPOSITION XIV.

A PROBLEM.

To describe a Circle about a regular Pentagon.



TO describe a Circle about the regular Pentagon $ABCDE$; divide equally two of its Sides AB and BC at G and H , and draw the Perpendiculars GF and HF . The Circle drawn from the Center F , at the Distance FA , will pass through B, C, D, E .

Demonstration.

Suppose the Circle described, it is evident (*by the 1. 3.*) that having divided the Line AB in the Middle in G , and drawn the Perpendicular GF , the Center of the Circle must be in that Perpendicular; it is also in HF , therefore it is at the Point F .

The USE

These Propositions are solely useful for the composing the Table of Sines, and drawing the Platforms of Citadels, for their ordinary Figures are Pentagons. Observe also, that these Methods of describing Pentagons about a Circle, may be applied likewise to other Polygons. But in my Book of Military Architecture,

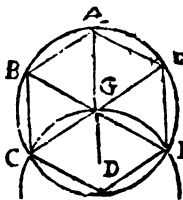
‘ texture, I have shown another Way of inscri-
‘ bing a regular Pentagon in a Circle.

PROPOSITION XV.

A PROBLEM.

To inscribe a regular Hexagon in a Circle.

TO inscribe a regular Hexagon in the Circle ABCDEF; draw the Diameter AD, and fixing the Foot of the Compass at the Point D, describe a Circle at the Distance of DG: then draw the Diameters EGB, and CGF; and the Lines AB, AF, FE, and the rest.



Demonstration.

’Tis evident, that the Triangles CDG, and DGE, are equilateral; therefore the Angles CGD, DGE, and those oppos’d to them at the Top, BGA, and AGF, are each of them the third Part of two right Angles; that is to say, contain 60 Degrees. Now all the Angles that can be made about the same Point are equal to four right Angles, that is to say, 360 Degrees. Therefore taking away four times 60, that is 240, from 360, there will remain 120 for BGC and FGE; which therefore each contain 60 Degrees. Therefore all the Angles, at the Center being equal, all the Arches and all the Sides will be equal; and every Angle, as A, B, C, &c. will be

compounded of two Angles of 60 Degrees each, that is, 120 Degrees, and therefore will be equal.

Coroll. The Side of a Hexagon is equal to the Semidiameter.

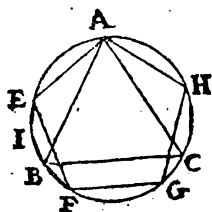
The U S E.

‘ Because the Side of a Hexagon is the Base of
‘ an Arch of 60 Degrees, and is equal to the
‘ Semidiameter, its Half will be the Sine of 30;
‘ by which Sine we begin the Table of Sines.
‘ *Euclid* treats of Hexagons in the last Book of
‘ his *Elements*.

PROPOSITION XVI.

A PROBLEM.

To inscribe a regular Pentedecagon in a Circle



I Nscribe in the Circle an equilateral Triangle ABC, (*by the 2.*) and a regular Pentagon, (*by the 11.*) so that the Angles may meet at the Point A. The Lines BF, BI, and IE, will be the Sides of a Pentedecagon; and if you inscribe in the other Arches Lines equal to BF and BI, you will complete the Polygon.

Demon-

Demonstration.

Since the Line AB is the Side of an equilateral Triangle, the Arch AEB will be the third Part of the whole Circle, that is, five fifteenths. But the Arch AE being the fifth Part will contain three fifteenths; therefore the Arch EB contains two: and if you divide it in the Middle at the Point I, each Part will be a Fifteenth.

The USE.

‘ This Proposition serves only to open the
 ‘ Way to other Polygons. We have in the
 ‘ *Compass of Proportion* some most easy Methods
 ‘ of inscribing all ordinary Polygons, but they
 ‘ are grounded on this here. For it would be im-
 ‘ possible to mark Polygons upon that Instru-
 ‘ ment, if their Sides were not first found by this,
 ‘ or other like Propositions.

THE FIFTH BOOK
OF THE
ELEMENTS
OF
EUCLID.

‘ **T**HIS fifth Book is absolutely neces-
 ‘ sary to demonstrate the Propositions
 ‘ of the sixth. The Doctrine it contains
 ‘ is of universal Use; and its Manner of Argu-
 ‘ mentation by proportion, the most subtil,
 ‘ solid and brief. Insomuch that all those Trea-
 ‘ tises, that are grounded on Proportions, are
 ‘ obliged to make use thereof, as a kind of
 ‘ *Mathematical Logick. Geometry, Arithmetick,*
 ‘ *Musick, Astronomy, Staticks,* and in a Word,
 ‘ all Parts of *Mathematicks*, are constrained to
 ‘ borrow some of their Demonstrations from
 ‘ the Propositions of this Book. The greatest
 ‘ Part of Measuring in *Practical Geometry* is
 ‘ done by Proportion. All the Rules of *Arith-*
 ‘ *metick* are demonstrable by the Theorems
 ‘ that

that occur here; so that there is no Necessity of having Recourse to the seventh, eighth or ninth Books for that purpose. The *Musick* of the Antients is scarce any Thing else but the Doctrine of Proportions apply'd to Sounds. The same may be said of *Statics*, which consider the proportion of Weights. In fine, 'tis most certain, that if the Knowledge of Proportions which this Book affords, was taken away from the *Mathematicks*, what remain'd would be very inconsiderable.

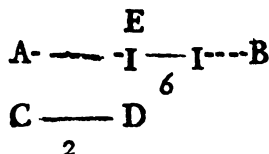
DEFINITIONS.

A Small Quantity compar'd with a greater, is call'd a Part. As for example,

if the Line CD, of 2 Foot long, be compar'd with AB of 6,

it will be call'd its *Part*. Which Name also it obtains, though indeed it be not contain'd in AB, provided AE, a Line equal to CD, be therein found.

The whole is answerable to the Part, and therefore will be the greater Quantity compar'd with the less; whether it do really contain the less, or not.



‘ A Part taken in general is ordinarily divided into (that which is call’d) an *Aliquot* Part, and an *Aliquant* Part.

1. An *Aliquot* Part (which alone *Euclid* defines in his Book) is a Magnitude of a Magnitude, a less of a greater, when it exactly measures the greater. ‘ That is to say, ‘Tis a lesser Quantity compar’d with a greater, which precisely measures the greater. As a Line two Foot long taken three Times, is equal to a Line of six Feet in length.

2. A *Multiple* is a Magnitude of a Magnitude, a greater of a less, when it is exactly measur’d by the less. ‘ That is to say, a Multiple is a greater Quantity compar’d with a less, which it contains exactly so many Times. ‘ For example, a Line six Foot long is Triple a Line two Foot long.

An *Aliquant* Part, is a lesser Quantity compar’d with a greater, which it does not exactly measure. As a Line of four Feet in length is an aliquant Part of a Line ten Foot long. ‘ In a Word, An aliquot Part so many Times repeated will equal the Whole: but an aliquant Part, though it contains such a Quantity of the Whole, yet repeated as you please, will never exactly equal, but either come short of, or exceed, the Whole.

Equimultiples are Magnitudes which equally contain their aliquot Parts. ‘ That is to say, ‘ so many Times. As for example, if A contains

12,	4;	6,	2.
A,	B;	C,	D.

contains B, as many Times as C contains D, A and C will be Equimultiples of B and D.

3. * *Proportion* is a respect of one Magnitude to another of the same Kind.

* *Græc.* λόγος. *Gall.* *Raison.*

4. *Quantities* are said to have a certain proportion to each other, when being multiply'd they can exceed each other. For which Reason they ought to be of the same Kind. For indeed a Line has no proportion to a Superficies; because a Line taken *mathematically*, is consider'd without any Breadth at all, and therefore multiply'd as much as you please, will never give any Breadth, which yet a Superficies contains.

For as much as Proportion is a Respect, or Relation founded upon Quantity, it ought to have two Terms. That which some Philosophers would call the *Fundamentum*, or Foundation, *Mathematicians* name the *Antecedent*, and the *Term* is call'd by them the *Consequent*. As if we were to compare the Quantity A with the Quantity B, that Respect or Proportion would have for the Antecedent the Quantity A, and for the Consequent the Quantity B. On the contrary, if B be compar'd with A, that proportion of B to A would have B for the Antecedent, and A for the Consequent.

Proportion, or Respect of one Quantity to another, is divided into Rational Proportion, and Irrational.

∴ Rati-

' Rational Proportion is the Respect of one
 ' Quantity to another, which is commensura-
 ' ble to it; that is, when both the Quantities
 ' have the same common measure, by which
 ' both may be exactly measur'd. As the pro-
 ' portion of a Line four foot long to another
 ' that is six, is rational, because a Line two
 ' Foot long may exactly measure both. And
 ' when this happens, these Quantities have
 ' the same Proportion as one Number to a-
 ' nother. For Example, since the Line two Foot
 ' long, which is their common Measure, is found
 ' twice in the four Foot Line, and thrice in
 ' that which is six Foot long; the first has the
 ' same Proportion to the second, as 2 to 3.

' Irrational Proportion is betwixt two Quan-
 ' tities of the same Kind, which are incommen-
 ' surable, *i. e.* have no common Measure. As
 ' the Proportion of the Side of a Square to its
 ' Diagonal. For there is no Measure so small, as
 ' will precisely measure both.

' Four Quantities will be *Proportionals*, when
 ' the proportion of the first to the second, is
 ' the same with, or like to, that of the third to
 ' the fourth; so that, to speak properly, Pro-
 ' portionality is a Similitude of Proportions.
 ' But it is no easy Matter to understand in what
 ' this Similitude of Proportions consists; that
 ' is to say, how two Respects or Relations may
 ' be alike. For *Euclid* has not given us a just
 ' Definition thereof, or such an one as might
 ' explain the Nature of the Thing, but content-
 ed

ed himself to set down some Marks or Signs,
by which it may be known, whether or no
Quantities have the same proportion. And
'tis the Obscurity of this Definition, which
has render'd the whole Book so difficult to be
understood; which Defect therefore I shall
endeavour to supply.

5. *Euclid* makes four Magnitudes to have the
same Proportion, when taking the Equimul-
tiples of the first and the third, and likewise
the Equimultiples of the second and the
fourth, according to any Multiplication what-
soever: If the Multiple of the first exceed
that of the second, the Multiple of the third
will also exceed that of the fourth; and if it
be equal to, or less than the second, the third
will be equal to, or less than the fourth. In
such a Case the first has the same proportion
to the second, as the third to the fourth.

A, B :: C, D	As for Example, if four
2, 4 :: 3, 6	Magnitudes were propos'd,
E, F :: G, H	A, B, C, D, having taken the
10, 8 :: 15, 12	Equimultiples of A and C;
K, L :: M, N	as their Quintuples E and G;
8 8 :: 12, 12	and F and H the doubles of
O, P :: Q R	B and D; In like manner ta-
6 16 :: 9 24	king K and M the Quadru-
	ples of A and C, and L and

N the Doubles of B and D; Again, taking O
and Q the Triples of A and C, and P and R the
Quadruples of B and D; because E being
greater than F, G is greater than H; and K
being

‘ being equal to L, M is equal to N; and lastly,
 ‘ O being less than P, Q is less than R: There-
 ‘ fore A will have the same proportion to B, as
 ‘ C to D. But methinks *Euclid* ought to have
 ‘ demonstrated this Proposition, it being too
 ‘ perplex’d and obscure to pass for a Definition.

‘ To explain aright what Proportionality is,
 ‘ or how four Magnitudes may be in the same
 ‘ Proportion; though it may be sufficient to say
 ‘ in general, that the first ought to be a like
 ‘ Part, or a like Whole in Respect of the second;
 ‘ as the third is, compar’d with the fourth;
 ‘ yet because this Definition agrees not to the
 ‘ proportion of Equality, I shall give a more
 ‘ general one; and to make it the more intelli-
 ‘ gible, explain first what is a similar, or like
 ‘ aliquot Part.

‘ Similar aliquot Parts, then, are such as are
 ‘ contain’d in their Wholes as many Times one
 ‘ as the other; as 3 in respect of 9 and 2 in
 ‘ respect of 6, are similar aliquot Parts; be-
 ‘ cause they are each contain’d three Times in
 ‘ their respective Wholes.

‘ The first Quantity will have the same pro-
 ‘ portion to the second, as the third to the fourth,
 ‘ if the first contains so many Times such ali-
 ‘ quot Parts of the second, as the third contains
 ‘ the like aliquot Parts of the
 ‘ fourth. As if A contains the | A, B, C, D.
 ‘ hundredth, thousandth or hun-
 ‘ dred-thousandth Part of B as oft as C contains
 ‘ the hundredth, thousandth, or hundred-thou-
 ‘ sandth

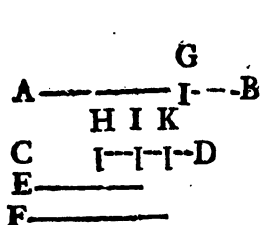
‘ tenth Part of E ; (and the like may be said
 ‘ of all other aliquot Parts imaginable :) There
 ‘ will be the same proportion of A to B , as of
 ‘ C to D .

‘ To render this Definition still more clear, I
 ‘ will prove first that, if A has the same pro-
 ‘ portion to B , as C to D , A will contain the
 ‘ aliquot Parts of B , as oft as C does the like
 ‘ of D ; and secondly, if A contain the aliquot
 ‘ Parts of B , as oft as C does the like of D , then
 ‘ there will be the same proportion of A to B , as
 ‘ of C to D .

‘ The first Point seems sufficiently evident
 ‘ from the very Notion of the Terms: for if A
 ‘ contains the tenth Part of B once more than
 ‘ an hundred Times, and C contains the tenth
 ‘ Part of D an hundred Times only: the Quanti-
 ‘ ty A will be a greater Whole compar’d to B ,
 ‘ than C is compar’d to D ; therefore it cannot
 ‘ be compar’d after the same manner, the Re-
 ‘ spect or Relation being not the same.

‘ The second Point seems more difficult, viz.
 ‘ if a Quantity, suppose AB , contain the aliquot
 ‘ Parts of another, CD , as oft as a third, E ,
 ‘ contains the like of the fourth, F ; there will
 ‘ be the same proportion of AB to CD , as of

‘ E to



‘ E to F. But if it be
 ‘ otherwise, let us sup-
 ‘ pose AB to have a great-
 ‘ er proportion to CD, than
 ‘ E has to F; that is to say,
 ‘ that AB is too great to
 ‘ have the same proporti-
 ‘ on to CD, that E has to F.
 ‘ Therefore a Quantity less than AB, as AG,
 ‘ will have the same proportion to CD, as E
 ‘ to F. Divide therefore CD in the Middle in
 ‘ H, and HD in the Middle in I, and ID in
 ‘ the Middle in K; continuing the like Division
 ‘ till you arrive at an aliquot Part of CD less
 ‘ than GB, which I will suppose to be KD.

Demonstration.

‘ Since there is the same proportion of AG to
 ‘ CD, as of E to F; AG will contain KD, an
 ‘ aliquot Part of CD, as oft as E contains the
 ‘ like aliquot Part of F. Now AB will con-
 ‘ tain KD once more than E contains the like
 ‘ aliquot Part of F; which is contrary, to the
 ‘ Supposition.

9. The first Quantity is said to have a greater
 proportion to the second, than the third to the
 fourth, when the first contains a certain ali-
 quot Part of the second, oftner than the third
 contains the alike aliquot Part of the fourth:
 ‘ As 101 has a greater proportion to 10 than 200
 ‘ to 20, because 101 contains the tenth Part
 ‘ of 10, that is, 1, once above a hundred Times;
 ‘ and

and 200 contains the tenth Part of 20, *i. e.* 2, a hundred Times only.

7. Magnitudes, or Quantities having the same proportion, are call'd Proportionals.

8 * Proportionality, or Analogy, is a Similitude of Proportions or Respects.

* *Ἀναλογία. Eucl.*

9. In each Proportionality are required at the least three Terms. 'For that there may be a Similitude of Proportions, there must be two of them: And every Proportion having two Terms, an Antecedent and a Consequent, there seems to be a Necessity of four Terms; as when we say, that A has the same proportion to B as C to D; but because the Consequent of the first proportion may be the Antecedent of the second, three Terms may suffice; as when A is said to have the same proportion to B as B to C.

10. Magnitudes are said to be continually proportional, when the intermediate Terms are taken twice, *i. e.* both as Antecedents and Consequents. 'As if there be the same proportion of A to B, as of B to C, and of C to D.

11. In that Case A will have the duplicate proportion to C, and the triplicate to D, of what it has to B.

'But here it is to observ'd, that there is a great deal of Difference between double Proportion, and duplicate. We say that the proportion of four to two is double, because four is the Double of two; the number two giving

the Name to the Proportion, or rather to the
 Antecedent thereof. Accordingly, double,
 triple, quadruple, quintuple, &c. are De-
 nominations drawn from the Numbers two,
 three, four, five, &c. compared with Unity;
 which Instance in, because the more easily
 we conceive the Proportion, the less are its
 Terms. But, as I said, these Denominations
 do rather affect the Antecedents, than the Pro-
 portions themselves; for we call that double,
 or triple Proportion, whose Antecedent is
 double, or triple its Consequent. But by
 duplicate Proportion we understand such an
 one, as is compounded of two similar Propor-
 tions. As, if there be the same proportion of
 two to four, as of four to eight: the proportion
 of two to eight being compounded of the
 Proportion of two to four, and that of four
 to eight, which are similar and equal, will be
 the duplicate of each of them. So three to
 twenty seven is the duplicate Proportion of
 that of three to nine. The proportion of two
 to four is call'd the Sub-double, because two
 is the half of four; but that of two to eight is
 the Duplicate of the Sub-double; which is as
 much as to say, that two is the Half of the
 Half of eight, as three is the third Part of
 the third Part of twenty seven; where you
 may observe, that the Denominators $\frac{1}{2}$ and $\frac{1}{3}$
 are taken twice.

In like manner, the proportion of eight to
 two is a duplicate proportion of that of
 eight

‘ eight to four, because eight is the Double of
 ‘ four, but it is the Double of the Double of two.
 ‘ If there be four Terms in continual propor-
 ‘ tion, the proportion of the first to the last is
 ‘ triplicate of that of the first to the second; as
 ‘ in these four Numbers, Two, Four, Eight,
 ‘ Sixteen; the proportion of two to sixteen is
 ‘ a triplicate of that of two to four, because
 ‘ two is the Half of the Half of the Half of
 ‘ sixteen. So also the proportion of sixteen to
 ‘ two is a Triplicate of that of sixteen to eight,
 ‘ because sixteen is the Double of the Double
 ‘ of the Double of two.

‘ 12. Antecedents to Antecedents, and Con-
 ‘ sequents to Consequents, are call’d Homolo-
 ‘ gous Magnitudes. As if there be the same pro-
 ‘ portion of A to B, as of C to D; A and C and
 ‘ D, are Homologous

‘ The following Definitions explain the di-
 ‘ vers Manners of arguing by Proportionals;
 ‘ for the Demonstration of which this Book
 ‘ was principally compos’d.

13. ‘ Alternate Proportion is when we com-
 ‘ pare the Antecedent of the one with the An-
 ‘ tecedent of the other, and the Consequent of
 ‘ the one with the Consequent of the other. As
 ‘ for example, if because there is the same pro-
 ‘ portion of A to B, as of C to D, I infer, that
 ‘ there is the same proportion of A to C, as of
 ‘ B to D. This manner of Argumentation holds
 ‘ only when all the four Terms are of the same
 ‘ species or Kind; i. e. either all Lines, or all

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‘ Superficies’s or all Solids. ’Tis demonstra-
 ‘ ted *Prop.* 16.

14. * Inverted Proportion is the comparing
 ‘ of the Consequents with the Antecedents.

* *Ἀντιστάσις*, *Eucl.* *Converse. Gall.*

‘ As, if because there is the same proportion
 ‘ of A to B, as of C to D, I conclude that there
 ‘ is the same proportion of B to A, as of D to
 ‘ C. *Coroll. of Prop.* 16.

15. Composition of Proportion is the com-
 ‘ paring of the Antecedent and the Consequent
 ‘ taken together, with the Consequent alone.

‘ As if, because there is the same proportion
 ‘ of A to B, as of C to D, I conclude that there
 ‘ is the same proportion of A and B to B, as of
 ‘ C and D to D. *Prop.* 18.

16. Division of Proportion is the comparing
 ‘ of the Excess of the Antecedent above the
 ‘ Consequent to the same Consequent. ‘ As, if
 ‘ there be the same Proportion of AB to B, as
 ‘ of CD to D; from thence I infer, that there
 ‘ is the same Proportion of A to B, as of C to
 ‘ D. *Prop.* 17.

17. Conversion of Proportion is the compa-
 ‘ ring of the Antecedent with the Difference
 ‘ of the Terms: ‘ As, if there be the same Pro-
 ‘ portion of AB to B, as of CD to D; I thence
 ‘ conclude that there is the same proportion
 ‘ of AB to A, as of CD to C. *Coroll. of Prop.*
 ‘ 18.

18. Proportion of Equality is the compa-
 ‘ ring of the extream Magnitudes, and omitting
 those

those in the Middle, ' As if there
 ' be the same Proportion of A to

A.	B.	C.	D.
E.	F.	G.	H.

 ' B, as of E to F, and of B to C,
 ' as of F to G; and of C to D, as of G to H;
 ' I infer, that there is the same proportion of
 ' A to D, as of E to H.

19. Proportionality of Equality orderly plac'd, is that in which the Terms are compar'd together in the same Order. ' As in the foregoing Example. *Prop.* 22.

20. Proportionality of Equality disorderly plac'd, is that in which the Terms are compar'd in a different Order. ' As if, there being the same Proportion of A to B, as of G to H; and of B to C, as of F to G; and of C to D, as of E to F; I conclude, that there is the same proportion of A to D, as of E to H. *Prop.* 23.

' See in short all the different Manners of Argumentation by Proportion.

- ' As A to B, so C to D; therefore,
 - ' By alternate Proportion: as A to C, so B to D.
 - ' Inverted. As B to A, so D to C,
 - ' Composition. As AB to B, so CD to D.
 - ' Division. If as AB to B, so CD to D. then as A to B, so C to D.
 - ' Conversion, As AB to A, so CD to C.
- Orderly Equality. If as A to B, so C to D; and as B to E, so D to F: then as A to E, so C to F.

Disorderly Equality. If as A to B, so D to F, and
as B to E, so C to D: then
as A to E, so C to F.

‘ *Euclid’s Fifth Book contains but 25 Propo-*
‘ *sitions, to which nine more have since been*
‘ *added, and are commonly receiv’d. And the*
‘ *first six in Euclid, serving only for the Proof*
‘ *of those that follow by the Method of Equi-*
‘ *multiples, since I intend not to make Use of*
‘ *that Method, I shall wholly omit; beginning*
‘ *with the Seventh, without changing either*
‘ *the Order or Number of the Propositions.*

Demands, or Suppositions.

Three Quantities, A, B, C, being propos’d, it
is required to be granted, that there is a fourth
possible, to which the Quantity C has the same
Proportion, as A to B.

PROPOSITION VII.

A THEOREM.

*Equal Quantities have the same Proportion to
another Quantity, and another Quantity has the
same Proportion to Quantities that are equal.*

IF the Quantities A and B be equal,
they will have the same Proportion
to the third C.

A, 8.
C, 4.
B, 8.

Demon-

Demonstration.

If one of the two, suppose A, had a greater Proportion to C, than B has to C; A would contain any aliquot Part of C, oftner than B could contain the same; and consequently A would be greater than B, which is contrary to what was suppos'd.

Again, I say, if the Quantities A and B be equal, the Quantity C will have the same Proportion to A as to B.

Demonstration.

If the Quantity C had a greater proportion to A, than to B, it would contain a certain aliquot Part of A, oftner than the like Part of B; which Part therefore of A must be less than the like aliquot Part of B, and consequently the Quantity A would be less than B, which is contrary to the Supposition.

PROPOSITION VIII.

A THEOREM.

The greater of two Quantities has a greater Proportion to the same, than the less; and the same Quantity has a lesser Proportion to the greater, than to the less.

A — I — B
 D

C — — —

GI

E — — — — — I — I — I — I — F

Suppose the Quantities AB and C be compar'd with the same EF, and that AB exceeds C. I say,

that AB will have a greater proportion to EF, than C will have to the same. Cut off AD equal to C, and divide EF in the Middle, and again one Half in the Middle, and so on, till you come to an aliquot Part of EF less than DB, as GF.

Demonstration.

AD and C being equal, AD has the same proportion to EF, as C to EF, (*by the 7.*) and therefore AD will contain GF an aliquot Part of EF, as oft as C will contain the same, (*by Defn. 5.*) But AB contains the same aliquot Part once more than AD, DB being greater than GF; therefore (*by Defn. 6.*) the proportion of AB to EF is greater than that of C to the same EF.

Secondly, I say, that EF has a less proportion to AB, than to C. Take a certain aliquot Part, as the fourth, of C, as oft as you can in EF, which

which suppose to be five times; either there will remain something of the Quantity EF, or nothing; if nothing remain, it is evident, that five times the fourth Part of AB making a greater Line than so many times the fourth Part of C, the fourth Part of AB could not be five times contain'd in EF. But if the fourth Part of C, taken five times reach no farther than G, the fourth Part of AB taken so many times will proceed either as far as F, or to I, something short of F. If it reach as far as F, EF will have the same proportion to AB, as EG to C. But (*by the preceding Part*) EF has a greater proportion to C, than EG to the same C; therefore EF has a greater proportion to C, than to AB. But if the fourth Part of AB reach no farther than I, EI will have the same proportion to AB as EG to C. But EI has a greater proportion to C, than EG to C; therefore EF greater than EI, has a greater proportion to C, than the same EF to AB.

PROPOSITION IX.

A THEOREM.

Quantities that have the same Proportion to another Quantity, or to which another Quantity has the same Proportion, are equal.

A, B, C, 12. 12. 6.	IF the Quantities A and B have the same proportion to a third Quantity C, I say, A and B are equal, <i>Demon-</i>
--------------------------------	---

Demonstration.

If one of the two, v. g. A were greater than B, it would have a greater proportion to the Quantity C, (*by the 1.*) which is contrary to the Supposition.

Secondly, if the Quantity C has the same proportion to the Quantities A and B; I say A and B are equal. For if A were greater than B, C would have a greater proportion to the Quantity B, than to A, (*by the 8.*) which is also contrary to the Supposition.

PROPOSITION X:

A THEOREM.

The Quantity that has the greater Proportion to another is the greater Quantity; and that the lesser, to which that other Quantity has the greater Proportion.

IF the Quantity A has a greater A. B. C. Proportion to the Quantity C, than B to the same C; I say A is greater than B. For if A and B, were equal, they would have both the same proportion to C; and if A were less than B, B would have a greater proportion to C, than A to the same C; both which are contrary to the Supposition.

Secondly, if C has a greater proportion to B than to A, I say that A will be greater than B.

B. For if A and B, were equal, C would have the same proportion to both, (*by the 7.*) And if A were less than B, C would have a less proportion to B than to A (*by the 8.*) both which are contrary to what was suppos'd.

PROPOSITION XI.

A THEOREM.

Proportions that are equal to another, are also equal amongst themselves.

[A, B; C, D; EF. **I** F A has the same proportion to B, as C to D, and C the same proportion to D, as E to F; I say that A will have the same proportion to B, as E to F.

4, 2; 8, 4; 6, 3.]

Demonstration.

Since A has the same proportion to B, as C to D; A will contain any certain aliquot Part of B, as oft 'as C contains the like aliquot Part of D, (*by Definit. 5.*) And in like manner as oft as C contains that aliquot Part of D, so oft will E contain the like aliquot Part of F. So that as oft as A contains any certain aliquot Part of B, so many times also will E contain the like aliquot Part of F. Therefore A has the same proportion to B, as E to F.

P R O-

PROPOSITION XII:

A THEOREM.

If many Quantities be proportional, one Antecedent will have the same Proportion to his Consequent, as all the Antecedents taken together to all the Consequents taken together.

IF A has the same proportion to B, as C to D; I say that A and C taken together will have the same proportion to B and D, as A to B.

A,	B,
3,	12,
C	D,
2,	8,

Demonstration.

Since A has the same proportion to B, as C to D; the Quantity A will contain any certain aliquot Part of B, as oft as C contains the like aliquot Part of D, (*by Defin. 5.*) suppose the fourth Part. Now the fourth Part of B and the fourth Part of D, make the fourth Part of DB; and accordingly AC will contain the fourth Part of BD, as oft as A contains the fourth Part of B; and the like may be said of any other aliquot Parts. Therefore A has the same proportion to B, as AC to BD.

P R O.

PROPOSITION XIII.

A THEOREM.

If of two equal Proportions one is greater than a third, the other will be so likewise.

[A, B : C, D : E F.] **I**F A has the same proportion to B, as C to D; but a greater proportion to B, than E to F: I say, that C also will have a greater proportion to D, than E has to F.

Demonstration.

Since A has a greater proportion to B than E to F, A will contain a certain aliquot Part of B, oftner than E contains the like aliquot Part of F, (*by Defn. 6.*) But C contains a like aliquot Part of D, as oft as A contains that of B; because A has the same proportion to B, as C to D: and therefore C contains a certain aliquot Part of D, oftner than E contains the like aliquot Part of F; and consequently, C has a greater proportion to D, than E to F, (*by Defn. 6.*)

PRO-

PROPOSITION XIV.

A THEOREM.

If the first Quantity has the same Proportion to the second, as the third to the fourth ; according as the first is greater, or equal, or less, than the third, the second will be greater, or equal, or less, than the fourth.

I F A has the same proportion to B, as C to D; I say first, if A be $\boxed{A, B : C, D}$ greater than C, B will be also greater than D.

Demonstration.

Since A is greater than C, A will have a greater proportion to B, than C to the same B, (*by the 8.*) But there is the same proportion of A to B, as of C to D: therefore C has a greater proportion to D, than C to B, and consequently (*by the 10.*) B is greater than D.

Secondly, if A be equal to C, B will be also equal to D.

Demonstration.

Since A and C are equal, there will be the same proportion of A to B, as of C to the same B, (*by the 7.*) But as A to B, so C to D; therefore C has the same proportion to B, as the same C to D, and consequently B and D are equal, (*by the 9.*)

Thirdly, if A be less than C, B will also be less than D.

Demon-

Demonstration.

Since A is less than C, A will have a less proportion to B, than C, to the same B, (*by the 8.*) But as A is to B, so C is to D: therefore C will have a less proportion to D, than the same C to B, and consequently B will be less than D, (*by the 10.*)

PROPOSITION XV.

A THEOREM.

Equimultiples, and similar Aliquot Parts, are in the same Proportion.

A	B;	C.	D,
2,	3;	6,	9,
E,	2;	H,	3,
F,	2;	I,	3,
G,	2;	K,	3

IF the Quantities C and D be the Equimultiples of A and B, their aliquot Parts; A will have the same proportion to B, as C to D. Divide the Quantity C into Parts equal to A, *v. g.* E, F, G, and the Quantity D into Parts equal to B. Because C and D are the Equimultiples of A and B, there will be as many Parts of one, as of the other. Let the Parts of D therefore be H, I, K.

Demonstration.

E has the same proportion to H, and F to I, and G to K, as A to B, because they are all equal. Therefore (*by the 12.*) as A to B, so E, F, G, to H, I, K, *i. e.* so C to D.

Coroll.

Coroll. The same Numbers of the aliquot Parts of two Quantities are in the same proportion that the Quantities are. For since E has the same proportion to H, as C to D; and F to I, as C to D; E and F will have the same proportion to H and I, as C to D.

P R O P O S I T I O N. XVI.

A T H E O R E M.

Alternate Proportion.

If four Magnitudes of the same Kind be proportional, they will be also alternatively so.

IF A has the same proportion to B as C to D, and all the four Quantities are of the same Kind, $A, B, C, D,$
 $12, 8, 9, 6,$ that is, either all Lines, or all Superficies's, or all Solids; A will have the same proportion to C, as B to D. For if not, suppose A to have a greater proportion to C, than B to D.

Demonstration.

Since 'tis suppos'd, that A has a greater proportion to C than B to D, the Quantity A will contain a certain aliquot Part of C, *v. g.* a third Part, oftner than B contains a third Part of D. Let A therefore contain a third Part of C four times, but B the third Part of D only three times; having then divided A into four Parts, each will contain one third Part of C; but B being

ing divided into four Parts, they will not contain each of them a third Part of D; therefore three Fourths of A will contain three Thirds of C, that is, the whole Quantity C; but three Fourths of B will not contain three Thirds of the whole Quantity D. But on the contrary, since there is the same proportion of A to B, as of C to D, there will be also the same proportion of three Fourths of A to three Fourths of B, as of C to D, (*by the Coroll. of the 15.*) and (*by the 14.*) if three Fourths of A be equal to C, three Fourths of B will be equal to D; therefore A cannot have a greater proportion to C, than B to D.

A L E M M A.

If the first have the same Proportion to the second, as the third to the fourth; any Aliquot Part of the first will have the same Proportion to the second, as the like Aliquot Part of the third to the fourth.

6,	3;	32,	6
A,	B;	C,	D
E,		F.	
4,		8.	

If A has the same proportion to B, as C to D; and E be an aliquot Part of A, and F a like aliquot Part of C: I say, that E will have the same proportion to B, as F to D.

Demonstration.

If E had a greater proportion to B than F to D, E would contain a certain aliquot Part of B, oftner than F contains the like aliquot Part of D; and consequently, E taken twice, thrice,

P

or

or four times, would contain an aliquot Part of B oftner than F, taken twice, thrice, or four times, contain'd the like aliquot Part of D. But E taken four times is equal to A; and likewise F taken four times is equal to C, therefore A would contain an aliquot Part of B, oftner than C contain'd the like aliquot Part of D, and by consequence A would have a greater proportion to B, than C to D; which is contrary to the Supposition.

A COROLLARY

which *Euclid* places after his 4th Proposit.

Inverted Proportion.

If the first has the same Proportion to the second, as the third to the fourth; the second will have the same proportion to the first, as the fourth to the third.

I F A has the same proportion to B, as C to D; B will have the same proportion to A, as D to C.

Demonstration.

A, B;	C, D,
4, 8;	12, 24,
E,	F,
1.	3.

If B had a greater proportion to A than D to C, B would contain an aliquot part of A; suppose a fourth E, oftner than D contain'd F the fourth Part of C. Let us suppose then that B contains eight times the Quantity E; D must contain but seven times the Quantity F. Now since A has the same proportion to B, as C to D, E will have the same proportion to B as F to D, (*by the preceding Lemma*.) and (*by the 15.*) E taken eight times will have the same

same proportion to B, as F taken eight times to D; but E taken eight times is contain'd in B, therefore F taken eight times must be contain'd in D, notwithstanding what was shewn to the contrary; therefore B cannot have a greater proportion to A, than D to C.

The USE.

' The Followers of *Averroes* seem to have
' made use of a manner of Argumentation not
' much unlike this, to prove, that *the World*
' *had existed from Eternity*; urging that there
' is the same proportion between an eternal
' Act of the Will of God, and the eternal Pro-
' duction of the World, as between a temporal
' Act, and a temporal Effect; therefore by a
' Kind of Alternation, there is the same pro-
' portion of a temporal Act of the Will, i. e.
' an Act beginning in Time, to an eternal Effect;
' as of an eternal Will to a temporal Effect. Now
' it is evident, that the Will, or an Act of the
' Will that begins in Time, cannot produce
' an eternal Effect; therefore the eternal Act of
' God could not produce an Effect in time. But
' this Argument is faulty in two Respects; first,
' in that it supposes it possible for an Act of
' Divine Will to begin in Time; and secondly,
' in that it is drawn from alternate Proportion,
' tho' the Terms be of a different Kind or Species.

PROPOSITION XVII.

A THEOREM.

The Division of Proportion.

If compounded Quantities be proportional, they will be so likewise being divided.

IF AB has the same proportion to B as CD to D, A will have the same proportion to B, as C to D.

AB, B; CD, D,
8, 3; 16, 9,
A, B; C, D,
5, 3; 10, 6,

Demonstration.

Since AB is suppos'd to have the same proportion to B as CD to D, AB will contain a certain aliquot Part of B, as oft as CD contains the like aliquot Part of D. Now that aliquot Part must be found as oft in B, as the like aliquot Part is found in D. Therefore taking away B from AB, and D from CD, A will contain as many aliquot Parts of B, as C contains the like of D; and consequently A will have the same proportion to B, as C to D.

PRO-

PROPOSITION XVIII.

A THEOREM.

The Compositions of Proportion.

If Quantities, being divided, be proportionable, they will be so likewise when compounded.

A, B, C, D, 5, 3; 10, 6. AB, B, CD, D, 8, 3; 16, 6,	I F A has the same proportion to B as C to D, AB also will have the same proportion to B as CD to D.
--	--

Demonstration.

Since A is suppos'd to have the same proportion to B as C to D, A will contain any aliquot Part of B, as oft as C contains the like aliquot Part of D. Now the Quantity B contains any of its own aliquot Parts, as oft as D contains the like of his; therefore adding B to A, and D to C, AB will contain any aliquot Part of B as oft as CD contains the like aliquot Part of D, and consequently (*by Defin. 5.*) AB will have the same proportion to B as CD to D.

A COROLLARY.

The Conversion of Proportion.

If AB has the same proportion to B as CD to D, then AB will have the same proportion to A as CD to C. For (*by the preceeding*) A has the same proportion to B, as C to D: and (*by the*

Coroll. of the 16.) B will have the same proportion to A, as D to C; and therefore compounding them, AB will have the same proportion to A, as CD to C.

The USE.

“ We have frequent Use of this manner of
 “ Argumentation in almost all Parts of the
 “ *Mathematicks.*

PROPOSITION XIX

A THEOREM.

If the Whole be in the same Proportion, as the Parts that are taken away from them, the Remainders will be also in the same Proportion.

IF the Quantity AB has the same proportion to CD, as the Part B to the Part D; I say, A will have the same proportion to C, as AC to CD.

AB, CD; B, D,
16, 8; 4, 2,
A, C; AB, CD,
12, 6; 16, 8,

Demonstration.

AB is suppos'd to have the same proportion to CD, as B to D; therefore alternatively (*according to the 16.*) AB has the same proportion to B, as CD to D; and by Conversion of Proportion, AB will have the same proportion to A, as CD to C; and again alternatively, there will be the same proportion of AB to CD, as of A to C.

The

The U S E.

‘ This Proposition is commonly made use of
 ‘ in the Rule of Fellowship. For instead of
 ‘ working by the Rule of Three for every par-
 ‘ ticular Associate or Partner, having done it for
 ‘ the rest, in the last they assign the Remainder
 ‘ of the Gain; supposing that if there be the
 ‘ same proportion of the whole Sum of all the
 ‘ Principals to the whole Gain, as of the Princi-
 ‘ pal of one Associate to his Part of Gain; there
 ‘ will be also the same proportion of the Princi-
 ‘ pal that remains to the Remainder of the Gain.
 ‘ The 20 and 21 Propositions are not ne-
 ‘ cessary.

PROPOSITION XXII.

A THEOREM.

• The Proportion of Equality orderly placed.

*If divers Terms be propos'd, and an equal Number
 of others compar'd with them, so that those
 which answer to each other in the same Order be
 proportional; the First and the Last will be also
 proportional*

IF the Quantities A, B, C, $12, 5, 2; 6, 3, 1,$
 and the Quantities D E, F, $A, B, C; D, E, F,$
 be proportional; that is, if
 there be the same proportion of A to B as of D

to E, and of B to C as of E to F; A will also have the same proportion to C, as D to F.

Demonstration.

If A has a greater proportion to C than D to F, A will contain an aliquot Part, v. g. the Half of C, oftner than D can contain the Half of F. Let us suppose then the Half of C to be contain'd twelve Times in A, and the Half of F only eleven Times in D. Now because B has the same proportion to C as E to F, the Quantity B will contain the Half of C, as oft as E contains the Half of F: Suppose then those Halves to be contain'd six Times in each, B and E. A, which contains the Half of C twelve times, will have a greater proportion to B, which contains the same Half of C six times; than D which contains the Half of F eleven times only, to E, which contains the same Half of F six times; and consequently A will have a greater proportion to B, than D to E, which is contrary to what was suppos'd.

PROPOSITION XXIII.

A THEOREM.

The Proportion of Equality disorderly plac'd

If two Orders of Terms, be in the same Proportion, disorderly plac'd: the first and the last of both Orders will be Proportional.

A, B, C, D, E, F, G,
12, 6, 3, 8, 4, 2, 1.

I F the Quantities, A, B, C, and the others D, E, F, equal in Number, be in the same

same proportion, disorderly plac'd, that is, if A has the same proportion to B, as E to F, and B to C, as D to E: A will have the same proportion to C, as D to F. Suppose B to have the same proportion to C, as F to G.

Demonstration.

Since there is the same proportion of A to B, as of E to F, and of B to C, as of F to G; A has the same proportion to C, as E to G, (*by the 22.*) Further, since B has the same proportion to C, as D to E, and also as F to G; D will have the same proportion to E, as F to G, (*by the 11.*) and alternatively, (*according to the 16.*) D will have the same proportion to F, as E to G. Now we have before prov'd, that as E to G, so A to C; therefore, as A to C, so D to F.

PROPOSITION XXIV.

A THEOREM.

If the first Quantity has the same Proportion to the second, as the third to the fourth; and also the fifth to the second, as the sixth to the fourth: the first with the fifth will have the same Proportion to the second, as the third with the sixth has to the fourth.

IF A has the same proportion to B, as C to D, and E to B, as F to D; AE will have the same proportion to B, as CF to D.

E,	F
4,	6.
6,	2, 9, 3.
A, B, C, D,	

De-

Demonstration.

Since A has the same proportion to B, as C to D, A will contain any aliquot Part of B, as oft as C contains the like aliquot Part of D, (by *Defin. 5.*) In like manner, E will contain the same aliquot Part of B, as oft as F contains the like of D; so that A and E will contain any aliquot Part of B, as oft as C and F contain the like aliquot Part of D; therefore AE will have the same proportion to B, as CF to D.

PROPOSITION XXV.

A THEOREM.

If four Magnitudes are proportional, the greatest and the least will exceed the other two.

B, D,
8, 5,
4, 3; 4, 3,
A, C : E, F.

IF the four Magnitudes AB, CD, E, F, be proportional, and AB the greatest and F the least; AB and F will exceed CD and E. Since AB has the same proportion to CD as E to F, and AB is suppos'd greater than E, CD will be also greater than F, (by the 14.) Divide therefore AB so, that the Magnitude A may be equal to E; and CD so, that the Magnitude C may be equal to F.

Demonstration.

Since AB has the same proportion to CD as A to C, B will also have the same proportion to D, as AB to CD, (by the 19) And AB being suppos'd

pos'd greater than CD, B will be greater than D. Now if to A and E, which are equal, be added F and C, which are also equal, A and F will be equal to C and E; and adding to the two first B, which is the greater, and to the two last D, which is the lesser, AB and F will be greater than CD and E.

The U S E.

' By this Proposition is demonstrated a Property of *Geometrical* Proportionality, where-
' by it is distinguished from that which is call'd
' *Arithmetical*. For in this latter the two mid-
' dle Terms are equal to the two Extreams;
' but in the former, (as has been prov'd,) the
' greatest and the least exceed the two others.

' Tho' the nine following Propositions are
' not *Euclid's*, yet, because many make use of
' them, and quote them as if they were his, I
' thought I ought not to omit them.

PRO-

PROPOSITION XXVI.**A THEOREM.**

If the first has a greater Proportion to the second than the third to the fourth, the fourth will have a greater Proportion to the third than the second to the first.

9, 4; 6, 3,
A, B; C, D,
E.
8.

IF A has a greater proportion to B than C to D, D will have a greater proportion to C than B to A. Suppose E to have the same proportion to B as C to D, A will be greater than E, (*by the 1c.*)

Demonstration.

There is the same proportion of E to B, as of C to D: therefore (*by the Coroll. of the 16.*) D has the same proportion to C, as B to E. But B has a greater proportion to E than to A, (*by the 8.*) therefore D has a greater proportion to C, than B to A.

PRO.

PROPOSITION XXVII.

A THEOREM.

If the first has a greater Proportion to the second than the third to the fourth, the first will also have a greater Proportion to the third than the second to the fourth.

IF A has a greater proportion to B than C to D, I say that A will have a greater proportion to C than B to D. Let E have the same proportion to B, as C to D; in that Case A must be greater than E.

9, 4; 6, 3,
A, B; C, D,
E.
8.

Demonstration.

E has the same proportion to B, as C to D; therefore (*by the 16.*) E has the same proportion to C, as B to D. And because A is greater than E, the proportion of A to C will be greater than that of E to C. Therefore the proportion of A to C, is greater than that of B to D.

PRO.

PROPOSITION XXVIII.

A THEOREM.

If the first has a greater Proportion to the second than the third to the fourth, the first and the second will have a greater Proportion to the second, than the third and the fourth to the fourth.

9, 4; 6, 3
A, B; C, D
E.
8.

IF the proportion of A to B be greater than that of C to D, the proportion of AB to B will also be greater than that of CD to D. Suppose E to have the same proportion to B, as C to D.

Demonstration.

E has the same proportion to B, as C to D: therefore (by the 18.) EB has the same proportion to B, as CD to D: And AB being greater than EB, AB will have a greater proportion to B, than EB to the same B, and consequently than CD to D.

PRO-

PROPOSITION XXIX.

A THEOREM.

If the first with the second has a greater Proportion to the second, than the third with the fourth to the fourth, the first will have a greater Proportion to the second, than the third to the fourth.

IF the proportion of AB to B be greater than the proportion of CD to D, the proportion of A to B will be also greater than that of C to D. Suppose EB to have the same proportion to B, as CD to D: then EB will be less than AB, and E less than A.

9.	4.	6.	3.
A,	B;	C,	D.
E.			
8.			

Demonstration.

Since EB has the same proportion to B, as CD to D; dividing them, E will have the same proportion to B as C to D, (by the 17.) And A being greater than E, the proportion of A to B will be greater than that of E to the same B, and consequently than that of C to D.

PRO-

PROPOSITION XXX.

A THEOREM.

If the Proportion of the first with the second to the second, be greater than that of the third with the fourth to the fourth ; the Proportion of the first with the second to the first, will be less than that of the third with the fourth to the third.

A, B ; C, D,
9, 4 ; 6, 3. **I**F the proportion of AB to B be greater than that of CD to D, the proportion of AB to A will be less than that of CD to C.

Demonstration.

The proportion of AB to B is suppos'd to be greater than that of CD to D: therefore (*by the 29.*) the proportion of A to B will be greater than that of C to D; and (*by the 26.*) the proportion of D to C will be greater than that of B to A; therefore being compounded (*by the 28.*) the proportion of CD to C, will be greater than that of AB to A.

PRO-

PROPOSITION XXXI.

A THEOREM.

If many Quantities are in a greater Proportion among themselves, than an equal Number of other Quantities, plac'd after the same manner; the first of the first Order will be in a greater Proportion to the last of that Order, than the first of the second Order to the last of that.

IF A has a greater proportion to B, than D to E; and B a greater proportion to C, than E to F; A will have a greater proportion to C, than D to F.

A, B, C, D, E, F,
16, 10, 3 19, 6, 2.

Demonstration.

Since A has a greater proportion to B than D to E, A will also have a greater proportion to D than B to E; and because B has a greater proportion to C than E to F, B will also have a greater proportion to E than C to F. Therefore A will have a greater proportion to D than C to F; and alternatively (*by the 27.*) A will have a greater proportion to C, than D to F.

PROPOSITION XXXII.

A THEOREM.

If many Quantities are in greater Proportion among themselves, than an equal Number of other Quantities, plac'd after a different manner; the first of the first Order will have a greater Proportion to the last of that Order, than the first of the second Order to the last of that Order.

13, 6, 2,		I F A has a greater proportion to C than I to K, and C a greater proportion to E than H to I; A will have a greater proportion to E, than H to K. Suppose B to have the same proportion to C as I to K, and C the same proportion to F as H to I; then A will be greater than B, and F than E.
A, C, E,		
B, F, H, I, K		
12, 3 4, 2, 1.		

H to K. Suppose B to have the same proportion to C as I to K, and C the same proportion to F as H to I; then A will be greater than B, and F than E.

Demonstration.

Since 'tis suppos'd that B has the same proportion to C as I to K, and C to F as H to I; B will have the same proportion to F, as H to K, (by the 23.) But A has a greater proportion to F, than B to the same F, (by the 8.) and the proportion of A to E is greater than that of A to F, because F is greater than E: therefore the proportion of A to E, is greater than that of H to K.

PRO-

PROPOSITION XXXIII.

A THEOREM.

If the Whole has a greater Proportion to the Whole than the Part taken away to the Part taken away, the Remainder will have a greater Proportion to the Remainder than the Whole to the Whole.

IF AB has a greater proportion to CD than B to D, A will have a greater proportion to C than AB to CD. 13, 3; 6, 2;
A, B; C, D,

Demonstration.

We suppose that the proportion of AB to CD is greater than that of B to D; therefore (by the 27.) the proportion of AB to B is greater than that of CD to D: and (by the 30.) the proportion of AB to A, is less than that of CD to C; therefore alternatively, the proportion of AB to CD is less than that of A to C.

Q. 2.

PRO-

Q. 2.

Q. 2.

PRO-

Q. 2.

Q. 2.

Q. 2.

PROPOSITION XXXIV.

A THEOREM.

If two Orders of Magnitudes be propos'd, and the Proportion of the first of the first, to the first of the second, be greater than that of the second to the second; and that greater than that of the third to the third, &c. the whole first Order will have a greater Proportion to the whole second, than the whole first Order except its first Magnitude, to the whole second Order except its first Magnitude: But a less Proportion than that of the first Magnitude of the first Order, to the first Magnitude of the second; and lastly, a greater Proportion than that of the last Magnitude of the first Order, to the last of the second.

12, 6.	4,	4, 3,	3,
A, B, C,	E,	F, G.	

IF the proportion of A to E be greater than that of B to F, and the proportion of B to F greater than that of C to G: I say, that A, B, C, will have a greater proportion to E, F, G, than the proportion of BC to FG.

Demonstration.

'Tis suppos'd the proportion of A to E is greater than that of B to F; and therefore alternatively, the proportion of A to B is greater than that of E to F; and by compounding them, the proportion of AB to B greater than that of EF to F; and again alternatively, the proportion

portion of AB to EF greater than that of B to F . And because the proportion of the Whole AB to EF is greater than that of the Part B to the Part F , the proportion of the Remainder A to the Remainder E , will be greater than that of the Whole AB to the Whole EF . In like manner, I may prove the proportion of B to F , greater than that of BC to FG , and consequently that of A to E , much greater than that of BC to FG . Therefore alternatively, the proportion of A to BC is greater than that of E to FG ; and compounding them, the proportion of A, B, C , to EC , greater than that of E, F, G , to FG : therefore the proportion of A, B, C , to E, F, G , will be greater than that of BC to FG .

Secondly, the proportion of A to E , is greater than that of A, B, C , to E, F, G .

Demonstration.

I have demonstrated, that the proportion of the Whole A, B, C , to the Whole E, F, G , is greater than that of the Part BC to the Part FG : therefore the proportion of the Remainder A to the Remainder E , will be greater than that of the Whole A, B, C , to the Whole, E, F, G , (by the 33.)

Thirdly, the proportion of A, B, C , to E, F, G , is greater than of C to G .

Demonstration.

The proportion of A to E is greater than that of B to F ; and therefore alternatively, that

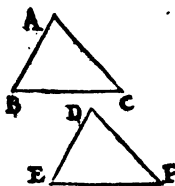
of A to B is greater than that of E to F ; and compounding them, the proportion of AB to B will be greater than that of EF to F ; and again alternatively, that of AB to EF will be greater than that of B to F . But the proportion of B to F is greater than that of C to G , therefore the proportion of AB to EF is greater than that of C to G ; and that of AB to C greater than that of EF to G ; and therefore by compounding them, the proportion of A, B, C , to G , will be greater than that of E, F, G , to G ; and that of A, B, C , to E, F, G , greater than that of C to G .



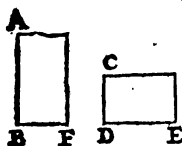
THE SIXTH BOOK
OF THE
ELEMENTS
OF
EUCLID.

THIS Book begins to apply the Doctrine of Proportions, explain'd in general in the preceding, to particular Matters; and taking its Rise from the most simple Figures, *i. e.* Triangles, it gives Rules to determine, not only the proportion of their Sides, but also that of their Capacity, *Area*, or Superficies. In the next Place we learn from it, how to find out proportional Lines, and to augment, or diminish any Figure, according to any proportion assign'd. Here also is demonstrat'd the most useful Rule of Three; and the Forty seventh of the First extended to any Figure whatsoever. Lastly, it lays down the most facile and most certain Principles to conduct us in taking any manner of Dimensions.

DEFINITIONS.



1. **R** Ectilinear Figures are similar, when their Angles are equal, and the Sides, that form those Angles, proportional. ' As the Triangles ABC, ' DEF, will be similar, if the ' Angles A and D, B and E, C ' and F, be equal; and AB has the same proportion to AC, as DE to DF; and AB to BC, as DE to EF.

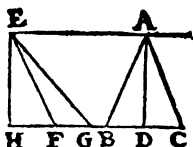


2. Figures are reciprocal, when they may be so compar'd, that the Antecedent of one Proportion and the Consequent of another are both found in the same Figure; That is, when the Analogy begins and ends in the same Figure. ' As if AB has the same proportion to CD, as ' DE to BF.

3. A Line is divided according to extreme and middle Proportion, when the whole Line has the same proportion to the greater Part, as the greater Part to the lesser. ' As if AB has the same proportion to AC, as AC ' to CB: the Line AB is divided according to ' extreme and middle Proportion.

4. The Height of any Figure is a Perpendicular drawn from its Summit to its Base.

As in the Triangles ABC, EFG, the Perpendiculars AD and EH, whether they fall within or without the Triangles, are their Heights. Hence it follows, that all Triangles and Parallelograms, that have equal Heights, may be plac'd within the same Parallels. For having set their Bases upon the Line HC, if the Perpendiculars DA and HE be equal, the Lines EA and HC will be Parallels.



5. A proportion is said to be compounded of many others, when the * Quantities of those proportions being multiply'd, make another. To understand the true Intent of this Definition, it must be observ'd, that every proportion, at least every † rational proportion, takes its Denomination from a certain Number, denoting that Respect or Relation that the Antecedent of the proportion bears to the Consequent. As if two Magnitudes were propos'd, one of twelve Foot in Length, and the other of six; we should call that proportion of 12 to 6 the double proportion. In like manner, if 4 and 12 were propos'd, we should give that the Name of subtriple proportion, $\frac{1}{3}$ being its Denominator; importing as much as, that the proportion of 4 to 12, is the same

* Denominators.

† Expressible by true Numbers.

' as that of $\frac{1}{3}$ to Unity, or as one to three.
 ' This Denominator is call'd the *Quantity* of
 ' the Proportion. Suppose therefore three
 ' Terms were given, 12, 6, and 2; the first
 ' proportion of 12 to 6 being double, its De-
 ' nominator is two; the second of 6 to 2 being
 ' Triple, its Denominator is three; the propor-
 ' tion therefore of 12 to 2 is said to be com-
 ' pounded of that of 12 to 6, and of 6 to 2,
 ' the double and the Triple proportions. To
 ' find therefore the Denominator of the pro-
 ' portion of 12 to 2, multiply three by two;
 ' and the Product 6 will show the proportion of
 ' 12 to 2 to be sextuple, *i. e.* as one is to six.
 ' This is that which *Mathematicians* commonly
 ' understand by compound Proportion, though
 ' methinks it might more properly have been
 ' call'd Proportion multiply'd.

PROPOSITION I.

A THEOREM.

*Parallelograms, and Triangles, of the same Height,
are in the same proportion as their Bases.*

Suppose the Triangles
 AGC and DEM, to
 have the same Height, and
 to be plac'd between the
 same Parallels, AD, and
 GM: I say, they will have



the

the same proportion as their Bases GC, and EM. Divide the Base EM into as many equal Parts as you please, and draw Lines from the Point D to each Division, as DF, DH, DL. In like manner divide the Line CG into Parts equal to those of the Line EM, and draw Lines from the Summit A to those Divisions, as AB, AI, &c. All those little Triangles, being inclos'd within the same Parallels, and having equal Bases, are equal, (*by the 38. 1.*)

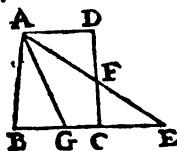
Demonstration.

The Base GC contains so many aliquot Parts of EM, as there are Parts found in it equal to EF; but as many Parts equal to EF as are found in the Base GC, so many little Triangles are contain'd in the Triangle AGC, equal to those contain'd in DEM; which being equal among themselves are the aliquot Parts of the Triangle DEM. As oft therefore as the Base GC contains those aliquot Parts of EM, so oft does the Triangle AGC contain the aliquot Parts of the Triangle DEM; which also will happen in every Division whatsoever: therefore the Triangle AGC has the same proportion to the Triangle DEM, as the Base GC to the Base EM.

Now Parallelograms, describ'd upon the same Bases, and inclos'd between the same Parallels; are double the Triangles, (*by the 41. 1.*) therefore they are in the same proportion as the Triangles, i. e. as their Bases.

The U S E.

‘ This Proposition is not only serviceable in
 ‘ demonstrating those that follow, but also of
 ‘ great use in dividing large Fields, or Plains.
 ‘ As for Example, suppose you were to take
 ‘ the third Part of the *Trapezium* ABCD, having the Sides
 ‘ AD and BC parallel; produce
 ‘ the Line BC to E, so that CE
 ‘ may be equal to AD; and tak-
 ‘ ing BG the third Part of BE,
 ‘ draw AG; I say the Triangle ABG is the third
 ‘ Part of the *Trapezium* ABCD.

*Demonstration.*

‘ The Triangles ADF, and FCE, are Equi-
 ‘ angular, because the Lines AD and CE are
 ‘ Parallels; and their Sides AD and CE are e-
 ‘ qual: therefore (*by the 26. 1.*) the Triangles
 ‘ are equal, and consequently the Triangle
 ‘ ABE, is equal to the *Trapezium* ABCD. But
 ‘ the Triangle AEG is the third part of the
 ‘ Triangle ABE, (*by the preceding*) therefore the
 ‘ Triangle ABG is the third Part of the *Trape-*
 ‘ *zium* ABCD.

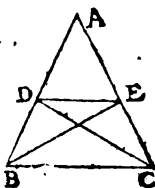
P R O.

PROPOSITION II.

A THEOREM.

A Line drawn in a Triangle parallel to its Base, divides its Sides proportionally; and the Line that divides the Sides of a Triangle proportionally, will be parallel to its Base.

IF in the Triangle ABC the Line DE be parallel to the Base BC, the Sides AB and AC will be divided proportionally; i. e. AD will have the same proportion to DB, as AE, to EC. Draw the Lines DC and BE. The Triangles DBE, and ECD, having the same Base DE, and being inclos'd within the same Parallels DE and BC, are equal, (*by the 37. 1.*)



Demonstration.

The Triangles ADE and DBE have the same Height E, and taking AD and DB for their Bases they may be plac'd within the Line AB, and another Parallel to it drawn through the Point E, and consequently have the same proportion as their Bases. (*by the 1.*) i. e. the Triangle ADE has the same proportion to the Triangle DBE, or its Equal CED, as AD to DB. But the Triangle ADE has likewise the same proportion to the Triangle CED, as AE to EC; and

Demonstration.

AB or EA has the same proportion to AC, as BD to DC : therefore EB and AD are parallel, and (by the 29. 1.) the alternate Angles EBA and BAD, as also the internal BEA and the External DAC, will be equal : and the Angles BEA and EBA being equal, the Angles BAD and DAC will be also equal. Therefore the Angle BAC will be divided into two equal Parts.

The USE.

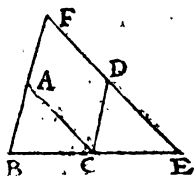
‘ We make Use of this Proposition chiefly
 ‘ to find the proportion of the Sides of Triangles.
 ‘ gles.

PROPOSITION IV.

A THEOREM.

The Sides of equiangular Triangles are proportional.

IF the Triangles ABC and DCE, be equiangular, i. e. if the Angles ABC and DCE, BAC and CDE be equal, AB will have the same proportion to BC as DC to CE, and also AC will have the same proportion to CB as DE to EC. Join the Triangles after such a manner, that the Bases BC and CE may be upon the same Line, and produce the Sides BA and ED



ED till they meet in F; since the Angles ACB and DEC are equal, the Lines AC and EF are Parallels, (*by the 28. 1.*) and by the same Reason CD and BF are Parallels, and therefore AFDC is a Parallelogram.

Demonstration.

In the Triangle BFE, FE is parallel to AC, therefore (*by the 2.*) AB has the same proportion to AF, or which is equal to it, CD, as BC to CE: and alternatively, AB has the same proportion to BC, as CD to CE. In like manner in the same Triangle, CD being parallel to BF, FD or AC has the same proportion to DE, as BD to CE, (*by the 2.*) and alternatively, AC has the same proportion to BC, as DE to CE.

The USE.

‘ This Proposition is of so general Use, that
 ‘ it may pass for a most universal Principle in
 ‘ taking all manner of Dimensions. For in the
 ‘ first place all the Methods of measuring inaccessible Lines, by describing a small Triangle
 ‘ similar to that which is form’d upon the
 ‘ ground, is founded upon it; as also the greatest
 ‘ Part of those *mathematical Instruments*, upon
 ‘ which are describ’d Triangles, similar to those
 ‘ of which we desire to take the Dimensions,
 ‘ as the *Geometrical Square*, the *Pantometer*, the
 ‘ *Arbalest* or *Cross-staff*, &c. Nor could we know
 ‘ how to raise the plane of any place, but by
 ‘ the Help thereof. So that in fine, to unfold
 ‘ all the Uses of this Proposition, it would be necessary

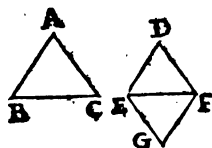
‘ necessary to transcribe the whole first Book of
‘ *Practical Geometry*.

PROPOSITION V.

A THEOREM.

Triangles, which have proportional Sides, are equiangular.

IF the Triangles ABC and DEF have proportional Sides, i. e. if AB has the same proportion to BC as DE to EF; and also AB the same proportion to AC as DE to DF; the Angles ABC and DEF, A and D, C and F, will be equal. Make the Angle FEG equal to the Angle B, and EFG equal to the Angle C.



Demonstration.

The Triangles ABC and EFG have two Angles equal, therefore (*by Coroll. 2. of the 32. 1.*) they are equiangular; and (*by the 4.*) AB has the same proportion to BC as GE to EF. Now 'tis suppos'd, that DE has the same proportion to EF as AB to BC; therefore DE has the same proportion to EF as EG to EF; and consequently (*by the 9. 5.*) DE and EG are equal. After the same manner DF may be prov'd equal to FG, and consequently (*by the 8. 1.*) the Triangles DEF and GEF are equiangular. But the

An-

Angle GEF was made equal to B, therefore the Angle DEF is equal to the Angle B, and the Angle DFE to the Angle C; and consequently, the Triangles ABC and DEF are equiangular.

PROPOSITION VI.

A THEOREM.

Triangles, which have each one of their Angles equal to one of the other, and the Sides containing that Angle Proportional, are equiangular.

IF the Triangles ABC and DEF [see fig. preced.] have the Angles B and E equal, and the Side AB has the same proportion to BC as DE to EF, the Triangles ABC and DEF will be equiangular. Make the Angle FEG equal to the Angle B, and the Angle EFG equal to the Angle C.

Demonstration.

The Triangles ABC and GEF are equiangular (by Coroll. 2. of the 32. 1.) therefore AB has the same proportion to BC, as GE to EF (by the 4.) But as AB to BC, so is DE to EF; therefore DE has the same proportion to EF as GE to the same EF; and therefore (by the 9. 5.) DE and EG are equal; and the Triangles DEF and GFE, having the Angles DEF and GEF, each equal to the Angle B, and the Sides DE and FG equal, with the Side EF common to both, will be equal in all Respects, (by the 4. 1.) therefore they

will be equiangular; and the Triangle EGF being equiangular to ABC, the Triangles ABC and DEF are equiangular.

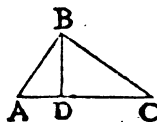
The Seventh Proposition is of no Use.

PROPOSITION VIII.

A THEOREM.

A Perpendicular, drawn from the right Angle of a rectangular Triangle to the opposite Side, divides the Triangle into two others similar to it.

IF the Perpendicular BD be drawn from the right Angle ABC to the opposite Side AC, it will divide the rectangular Triangle ABC into two Triangles ADB and BDC, which will be similar or equiangular to the Triangle ABC.



Demonstration.

The Triangles ABC and ADB have the same Angle A, and the Angles ABC and ADB right Angles, therefore they are Equiangular, (*by Coroll. 2. of the 32. 1.*) In like manner, the Triangles BDC and ABC have the Angle C common to both, and the Angles ABC and BDC right Angles; therefore they also are equiangular. Therefore the Triangles ABC, BDC, and ABD, are similar Triangles.

The

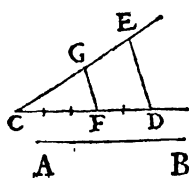
The USE.

By the Help of this Proposition inaccessible Distances may be measur'd by a Carpenter's Square. As for Example, if I were to measure the Distance DC; having drawn the perpendicular BD, and plac'd my Square upon the Point B in such a manner, that by one of its Sides BC I could observe the Point C, and by the other the Point A: 'tis evident, there would be the same proportion of AD to DB, as of DB to DC. Therefore multiplying DB by itself, and dividing the Product by AD, the Quotient would be DC.

PROPOSITION IX.

A PROBLEM.

From a Line given to cut off what Part you please.



LET the Line propos'd be AB, from which you desire to take away 3 fifth Parts. Make an Angle ECD, and upon one of its Sides CD take five equal Parts, three of which shall be contain'd in CF: then taking CE equal to AB, draw the Line DE, and another parallel to that FG: the Line CG will contain three fifth Parts of CE, or AB.

R 3

Demon-

Demonstration.

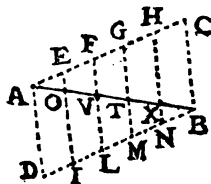
In the Triangle ECD, FG being parallel to the Base DE, CF will have the same proportion to FD as CG to GE, (*by the 2.*) and compounding them (*by the 18. 5.*) CE will have the same proportion to CG as CD to CF; and (*by the Coroll. of the 16. 5.*) CG will have the same proportion to CE as CF to CD. But CF contains three Fifths of CD, therefore CG will contain three Fifths of CE or AB.

PROPOSITION X.

A PROBLEM.

To divide a Line after the same manner as another Line given is divided.

IF you would divide the Line AB after the same manner as AC is divided, make with the two Lines the Angle CAB of what Magnitude you please: then draw the Line BC, and parallel to it the Lines EO, FV, and the rest. The Line AB will be divided after the same manner that AC is.

*Demonstration.*

Since in the Triangle BAC, HX is drawn parallel to the Base BC, it will divide the Sides
AB

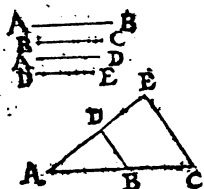
AB and AC proportionally (*by the 2.*) and the same may be said of all the rest.

To do this more easily you may draw the Line BD parallel to AC; and transferring the Divisions of AC, to BD, draw Lines from one to the other.

PROPOSITION XI.

A PROBLEM.

Two Lines being given to find a third Proportional.



IF you would find a third Proportional to the Lines AB and BC, *i. e.* that there may be the same Proportion of AB to BC, as of BC to the Line sought; make at pleasure the Angle EAC, and upon one of its Sides take the Lines AB and BC, one immediately after the other; and upon the other Side take AD equal to BC: then draw the Line BD, and parallel to it the Line CE; and the Line DE will be that which you seek.

Demonstration.

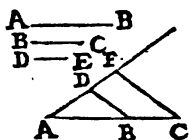
In the Triangle EAC the Line DB is parallel to the Base CE, therefore (*by the 2.*) there is the same proportion of AB to BC, as of AD or BC to DE.

PROPOSITION XII.

A PROBLEM.

Three Lines being given to find a fourth Proportional.

LET the three Lines propos'd, to which you are to find a fourth Proportional, be AB, BC, and DE. Make at pleasure the Angle FAC, and take upon AC the Lines AB and BC, and upon AF the Line AD equal to DE; then draw the Line DB, and parallel to it FC: I say, that DF is the Line sought, *i. e.* there is the same proportion of AB to BC as of DE or AD to DF.

*Demonstration.*

In the Triangle FAC the Line DB is parallel to the Base FC; there is therefore the same proportion of AB to BC, as of AD to DF, (*by the 2.*)

The USE.

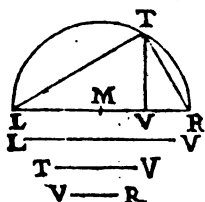
‘ The Use of the *Compass* of *Proportion* is
 ‘ grounded upon these four Propositions, for
 ‘ that Instrument teaching us to divide a Line
 ‘ as we please; to make use of the *Rule of Three*
 ‘ without the Help of *Arithmetick*; to extract
 ‘ the square, and cubick Roots; to double the
 ‘ Cube; to measure all sorts of Triangles; to
 ‘ find the Capacity of Superficies’s, and the So-
 ‘ lidity

‘ lidity of Bodies ; and to augment or diminish
 ‘ any Figure, according to what proportion we
 ‘ desire ; all these Operations are demonſtrated
 ‘ by the preceding Propoſitions.

PROPOSITION XIII.

A PROBLEM.

Two Lines being given to aſſign a middle Proportional.



IF you deſire a middle Proportional between the Lines LV, and VR ; having plac'd them ſo, that they make but one right Line LR, divide that Line into two equal Parts in M ; and having deſcrib'd a Semicircle LTR from the Center, draw the Perpendicular VT, which will be a middle Proportional between LV and VR. Draw the Lines LT, and TR.

Demonſtration.

The Angle LTR, being deſcrib'd in a Semicircle, is a right Angle, (*by the 31. 3.*) and (*by the 8.*) the Triangles LVT and TVR are ſimilar ; therefore there is the ſame proportion of LV to VT in the Triangle LVT, as of VT to VR in the Triangle TVR, (*by the 4*) therefore VT is a middle Proportional between LV and VR.

The

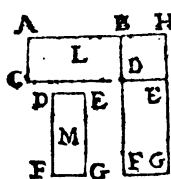
The U.S.E.

‘ By this Proposition any rectangular Parallelogram may be reduc’d to a Square. For Example, in the Rectangle contain’d under LV and VR, the Square of VT is equal to the Rectangle under LV and VR; as I shall hereafter demonstrate.

PROPOSITION. XIV.

A THEOREM.

Equal equiangular Parallelograms have their Sides reciprocal; and equiangular Parallelograms, which have their Sides reciprocal, are equal.



IF the Parallelograms L and M be equiangular and equal, their Sides will be reciprocal, *i. e.* CD will have the same proportion to DE as FD to DB. Since they have equal Angles, they may be so join’d together, that the Sides CD and DE, FD and DB, will concur in two right Lines, (*by the Coroll. of the 15. 1.*) producing therefore the Sides AB and GE, you complete the Parallelogram BDEH.

Demonstration.

The Parallelograms L and M being equal, will have the same proportion to the Parallelogram BDEH: But the proportion of L to BDEH, is

as the Base CD to the Base DE; and that of M, or DFGE, to BDEH, is as the Base FD to the Base DB (*by the 1.*) Therefore CD has the same proportion to DE, as FD to DB.

Secondly, if the Parallelograms L and M be equiangular, and have their Sides reciprocal, they will be equal.

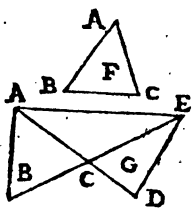
Demonstration.

The Sides of the Parallelograms are suppos'd to be reciprocal, *i. e.* there is the same proportion of CD to DE, as of FD to DB: but as the Base CD is to DE, so is the Parallelogram L to the Parallelogram BDEH (*by the 1.*) and as FD to DB, so is the Parallelogram M to BDEH; therefore L has the same proportion to BDEH, as M to the same BDEH; therefore (*by the 9. 5.*) the Parallelograms L and M are equal.

PROPOSITION XV.

A THEOREM.

Equal Triangles that have one Angle equal each to the other, have the Sides that form that Angle reciprocal. And if those Sides be reciprocal, they will be equal.



IF the Triangles F and G, being equal, have the Angles ACB and DCE equal, their Sides that form those Angles, will be reciprocal; *i. e.* BC will have the same proportion to GE as CD to CA. Place the Triangles so, that the Sides CD and CA may make one right Line; and then because the Angles ACB and DCE are suppos'd to be equal, BC and CE will also make one right Line, (*by Coroll. of the 15. 1.*) Draw the Line AE.

Demonstration.

The Triangle ABC has the same proportion to the Triangle ACE, as the Triangle ECD equal to the former, to the same ACE, (*by the 7. 5.*) But as ABC to ACE, so is the Base BC to the Base CE, (*by the 1.*) having both the same Height; and as ECD to ACE, so is the Base CD to CA, (*by the same* :) therefore BC has the same proportion to CE, as CD to CA. But if the Sides be suppos'd reciprocal, *i. e.* that BC has the

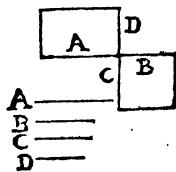
the same proportion to CE, as CD to CA, the Triangles ABC and CDE will be equal, because they will both have the same proportion to ACE.

PROPOSITION XVI.

A THEOREM.

If four Lines be proportional, the Rectangle contain'd under the first and the fourth, will be equal to the Rectangle contain'd under the second and the third. And if the Rectangle contain'd under the Extrems be equal to that contain'd under the middle Terms, the four Lines will be proportional.

IF the Lines A, B, C, D, be proportional, *i. e.* if as A to B, so C to D, the Rectangle contain'd under the first A, and the fourth D, will be equal to the Rectangle contain'd under B and C.



Demonstration.

The Rectangles have one Angle equal each to the other, because 'tis a right Angle in both; their Sides also are reciprocal: therefore they are equal, (*by the 14.*)

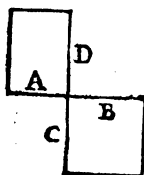
In

In like manner, if they are equal, their Sides will be reciprocal, *i. e.* A will have the same proportion to B, as C to D.

PROPOSITION XVII,

A THEOREM.

If three Lines be proportional, the Rectangle contain'd under the first and the third, will be equal to the Square of the middle Term. And if the Square of the middle Term be equal to the Rectangle under the Extreams, the three Lines are proportional.



IF the three Lines A, B, D, be proportional, the Rectangle contain'd under A and D will be equal to the Square of B. Take C equal to B, and there will be the same proportion of A to B, as of C to D; therefore the four Lines are proportional.

Demonstration.

The Rectangle under A and D will be equal to that under B and C, (*by the preceding,*) but the last Rectangle is a Square (the Lines B and C being equal) therefore the Rectangle contain'd under A and D is equal to the Square of B.

In

In like manner, if the Rectangle under A and D be equal to the Square of B, A will have the same proportion to B as C to D: and B and C being equal, A will have the same proportion to B, as B to D.

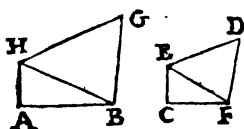
The U S E.

By these four Propositions may be demonstrated that Rule in *Arithmetick*, which is commonly call'd the *Rule of Three*; and consequently, the Rules of *Fellowship*, of *False*, and all those others that depend upon proportion. For Example, suppose three Numbers given, A 8, B 6, and C 4, and it be requir'd to find a fourth proportional Number; which taking as found, I will call D. The Rectangle then contain'd under A and D, is equal to that under B and C. But I may have this latter Rectangle by multiplying B by C, i. e. Six by Four; the Product will be Twenty-four; therefore the Rectangle contain'd under A and D is also twenty four; and therefore dividing that Number by A, which is 8, the Quotient Three will be the Number sought.

PROPOSITION XVIII.

A PROBLEM.

To describe a Polygon similar to another upon a Line given.



LET AB be the Line assign'd, upon which you are required to describe a Polygon similar to the Polygon CFDE; having divided the Polygon CFDE into Triangles, upon the Line AB make a Triangle ABH similar to the Triangle CFE, *i. e.* make the Angle ABH equal to the Angle CFE, and BAH equal to FCE: for then the Triangles ABH and CFE will be equiangular, (*by Coroll. 2. of the 32. 1.*) Make also upon the Line BH a Triangle equiangular to FDE.

Demonstration.

Since the Triangles, which are Part of the Polygons are equiangular, the two Polygons are equiangular. Further, since the Triangles ABH and CFE are equiangular, AB will have the same Proportion to BH as CF to FE, (*by the 4.*) In like manner, the Triangles HBG and FED being equiangular, BH will have the same Proportion to BG as EF to FD: and by Equality,
(accord-

(according to *Defn.* 18. 5.) AB will have the same Proportion to BG, as CF to FD. And the same may be said of all the other Sides. Therefore (by *Defn.* 1.) the Polygons are similar.

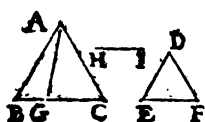
The U S E.

‘ Upon this Proposition is grounded the
 ‘ greatest Part of *Practical Geometry*, that re-
 ‘ lates to the raising the Plan of any Place, as
 ‘ of a Building, Field, Forest, or a whole
 ‘ Country: For having divided a Line into e-
 ‘ qual Parts, to answer the Feet or Yards con-
 ‘ tain’d in the Plan, you may describe a Figure
 ‘ similar to, but less than, the Original, in
 ‘ which you may see the proportion of all its
 ‘ Lines. And having by Experience found it
 ‘ much more easie to travel upon Paper, than
 ‘ to take a tedious Journey either by Land or
 ‘ Water, this Proposition will likewise afford
 ‘ us Assistance in this respect; informing us in
 ‘ almost all the Parts of *Geodesia*, and *Chorogra-*
 ‘ *phy*; and giving Instructions how to compose
 ‘ *Geographical Charts*, and *Maps*; which are
 ‘ nothing else but Methods of reducing great
 ‘ Figures to small. Further, the Use of this
 ‘ Proposition extends its self to almost all those
 ‘ Arts, that require the Design and Model of
 ‘ their Works before-hand.

PROPOSITION XIX.

A THEOREM.

Similar Triangles are in the duplicate Proportion of their homologous Sides.



IF the Triangles ABC and DEF be similar, or equiangular, they will be in the duplicate proportion of their homologous Sides BC, EF, *i. e.* the proportion of the Triangle ABC to the Triangle DEF will be the Duplicate of the proportion of BC to EF; so that finding a third Proportional HI to the Lines BC and EF, and making BC to have the same proportion to EF, as EF to HI, the Triangle ABC will have the same proportion to DEF as the Line BC to the Line HI; which is to have to it a duplicate proportion (*by Defn. 11. 5.*) Take BG equal to HI, and draw the Line AF.

Demonstration.

The Angles B and E of the Triangles ABG and DEF are equal; and besides, since the Triangles ABC and DEF are similar, AB will have the same proportion to DE as BC to EF, (*by the Fourth.*) But as BC to EF, so EF to HI or BG; therefore as AB to DE, so EF to BG; and consequently, the Sides of the Triangles ABG and DEF being reciprocal, the Triangles will

will be equal, (*by the 15.*) And (*by the 1.*) the Triangle ABC has the same proportion to the Triangle ABG, as BC to BG or HI; therefore the Triangle ABC has the same proportion to the Triangle DEF, as BC to HI.

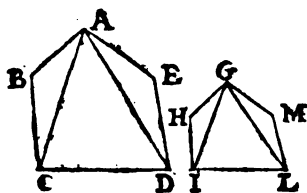
The USE.

‘ These Propositions may help to correct the
‘ Error of those, who art apt to imagine similar
‘ Figures to have the same proportion as their
‘ Sides. For if two Squares, two Pentagons,
‘ two Hexagons, or two Circles, be propos’d,
‘ and the Side of the first be double that of the
‘ second, the first Figure will be quadruple the
‘ second, if the Side of the first be triple that
‘ of the second, the first Figure will be nine
‘ times greater than the second. Therefore to
‘ make a Square triple to another, you must
‘ seek a middle proportional between one and
‘ three, and you ll find for the Side of your
‘ triple Figure almost $1\frac{1}{4}$.

PROPOSITION XX.

A THEOREM.

Similar Polygons may be divided into an equal Number of Triangles, and are in the duplicate Proportion of their homologous Sides.



IF the Polygons
ABCDE and
GHILM be similar,
they may be divided
into an equal Number
of similar Triangles,
which will be the si-

imilar Parts of their Wholes. Draw the Lines
AC, AD, GI, GL.

Demonstration.

Since the Polygons are similar, their Angles
B and H will be equal: and AB will have the
same proportion to BC as GH to HI, (*by Defn.*
1.) therefore the Triangles ABC and GHI are si-
milar, (*by the 6.*) and (*by the 3.*) BC has the same
proportion to CA as HI to GI. Further, be-
cause CD has the same proportion to BC as IL
to IH, and BC the same to CA as HI to IG; by
Equality, CD will have the same proportion to
CA as IL to GI. Now the Angles BCD and AIL
being equal, if the Angles ACB and GIH, which
are equal, be taken from them, the Angles
ACD and GIL will remain equal. Therefore
(*by*

(*by the 6.*) the Triangles ACD and GIL will be similar. In like manner, 'tis easie to run over all the Triangles of the Polygons, and to prove them similar.

I add further, that the Triangles are in the same proportion as the Polygons.

Demonstration.

Since all the Triangles are similar, their Sides will be proportional, (*by the 4.*) but each Triangle to its similar is in the duplicate proportion of their homologous Sides, (*by the 19.*) therefore every Triangle of one Polygon to every Triangle of the other, is in the duplicate proportion of their Sides; which being the same, the duplicate proportion must be the same; and there will be the same proportion of each Triangle to its similar, as of all the Triangles of one Polygon to all the Triangles of the other Polygon, (*by the 12. 5.*) *i. e.* of one Polygon to the other.

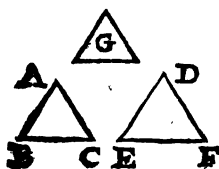
Coroll. 1. Similar Polygons are in the duplicate proportion of their homologous Sides.

Coroll. 2. If three Lines be in continual proportion, a Polygon describ'd upon the first will have the same proportion to a Polygon describ'd upon the second, as the first Line to the third, *i. e.* it will be in the duplicate proportion of that of the first Line to the second.

PROPOSITION XXI.

A THEOREM.

Polygons, that are similar to another Polygon, are so also amongst themselves.



IF two Polygons be similar to a third, they will be so also betwixt themselves. For they may each be divided into as many similar Triangles, as are in the third. But Triangles similar to a third, are also similar amongst themselves; because Angles equal to a third, are equal amongst themselves; and the Angles of the Triangles being equal, those of the Polygons being compounded of them, must be so likewise.

I add, that the Sides of the Triangles being proportional, those of the Polygons must be so also, because they are the same.

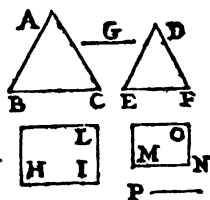
P R O.

PROPOSITION XXII.

A THEOREM.

Similar Polygons describ'd upon four proportional Lines, are also proportional. And if the Polygons be proportional, the Lines are so too.

IF BC has the same proportion to EF as HI to MN, the Polygon ABC will also have the same proportion to the similar Polygon DEF, as HL to its similar Polygon MO. Seek a third Proportional G



to the Lines BC and EF, and to the Lines HI and MO another third Proportional P, (*by the 11.*) Since BC has the same Proportion to EF as HI to MN, and EF to G as MN to P; by equality, BC will have the same proportion to G, as HI to P: and this proportion will be the Duplicate of that of BC to EF, or HI to MN.

Demonstration.

The Polygon ABC to the Polygon DEF is in the duplicate proportion of that of BC to EF; (*by the 2c.*) that is, as BC to G; and the Polygon HL has the same proportion to MO, as HI to P. Therefore ABC has the same proportion to DEF, as HL to MO.

And if the similar Polygons be proportional, the Lines being in the subduplicate proportion to them, will be also proportional.

The USE.

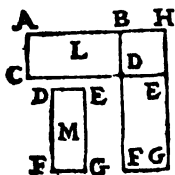
A, B; C, D,
3, 2; 6, 4,
9, 4; 36, 16,
E, F; G, H,

' This Proportion may be easily apply'd to Numbers, if the Numbers A, B, C, D, be proportional, their Squares E, F, G, H, will be so too; which is very serviceable in *Arithmetick*, and more in *Algebra*.

PROPOSITION XIII.

A THEOREM.

Equiangular Parallelograms are in the Proportion compounded of the Proportions of their Sides.



IF the Parallelograms L and M be equiangular, the proportion of L to M will be compounded of that of AB to DE, and that of BD to DF. Joyn the Parallelograms so that their Sides BD and DF may make but one right Line, as also CD and DE another; which, the Parallelograms being equiangular, may be done (*by the Coroll. of the 15. 1.*) and complete the Parallelogram BDEH.

Do

Demonstration.

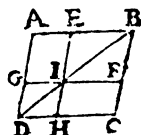
The Parallelogram L has the same proportion to the Parallelogram BDEH, as the Base AB to the Base BH or DE, (*by the 1.*) and the Parallelogram BDEH has the same proportion to the Parallelogram DFGE, *i. e.* M, as the Base BD to DF. But the proportion of the Parallelogram L to the Parallelogram M, is compounded of that of L to the Parallelogram BDEH, and of that of BDEH to the Parallelogram M. Therefore the proportion of L to M is compounded of that of AB to DE, and that of BD to DF. For Example, let AB be 8, DE 5, BD 4, DF 7; and make as 4 to 7, so 5 to $8\frac{1}{4}$; by which means you will have three Numbers, 8, 5, and $8\frac{1}{4}$, 8 to 5 being the proportion of the Parallelogram L to BDEH, which is that of AB to DE; and 5 to $8\frac{1}{4}$ that of the Parallelogram BDEH to M. Taking away therefore the middle Term five, there will remain 8 to $8\frac{1}{4}$ for the proportion compounded of the two

PROPOSITION XXIV.

A THEOREM.

In all Parallelograms, those through which the Diameter passes, are similar to the great one.

SUPPOSE the Diameter of the Parallelogram AC pass thro' the Parallelograms EF, GH; I say they are similar to the Parallelogram AC.



Demon-

Demonstration.

The Parallelograms AC and EF, have the same Angle B, and because in the Triangle BCD, IF is parallel to the Base DC, the Triangles BFI and BCD are equiangular. Therefore (by the fourth) BC has the same Proportion to CD as BF to FI, and consequently the Sides are in the same proportion. In like manner IH being parallel to BC, DH will have the same proportion to HI as DC to BC; the Angles are also equal, all the Sides being Parallels: therefore (by Defn. 1.) the Parallelograms EF and GH are similar to the Parallelogram AC.

The USE.

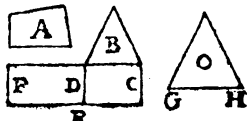
‘ I have made use of this Proposition to demonstrate the 10 Proposition of my last Book of *Perspectives*; where I have shown a way to draw an Image similar to the Original, by a Parallelogram compos’d of four Rulers.

PROPOSITION XXV.

A PROBLEM.

To describe a Polygon, similar to one given, and equal to another.

IF you desire to describe a Polygon equal to the Rectilineal A, and similar to the Polygon B; make a Parallelogram CE equal to



the

the Polygon B, (*by the 44. 1.*) and upon the Line DE make another Parallelogram equal to the Rectilineal A, (*by the 45. 1.*) Then find a middle proportional GH between CD and DF, (*by the 13.*) Lastly, make upon GH a Polygon O, similar to B, (*by the 18.*) which will be equal to the Rectilineal A.

Demonstration.

Since CD, GH, and DF, are in a continual proportion, the Rectilineal B describ'd upon the first, will have the same proportion to the Rectilineal O describ'd upon the second, as CD to DF, (*by Coroll. 2. of the 20.*) But as CD to DF, so is the Parallelogram CE to FE, or B to A, which are equal to them. Therefore B has the same proportion to O as B to A, and consequently (*by the 9. 5.*) A and O are equal

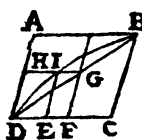
The USE.

‘ This Proposition teaches how to change one
 ‘ Figure into another, retaining still its equal-
 ‘ lity to a third; which is very useful in *Prac-*
 ‘ *tical Geometry*, for the reducing all Figures to
 ‘ Squares,

PROPOSITION XXVI.

A THEOREM.

If in one Angle of a Parallelogram you describe a less, similar to the former, the Diameter of the greater will fall upon the Angle of the less.



I F in the Angle D of the Parallelogram AC you describe a lesser Parallelogram DG, similar to the other, the Diameter BD will pass by the Point G. For if it do not pass by that Point, suppose it then to pass by the Point I, and to make the Line BID. Draw the Line IE parallel to HD.

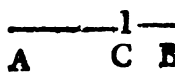
Demonstration.

The Parallelogram DI would be similar to the Parallelogram AG (*by the 24.*) But the Parallelogram DG is also suppos'd similar to it, therefore the Parallelograms DI and DG would be similar, which is impossible; for if so, HI would have the same proportion to IE or GF, as HG to the same GF; and (*by the 6. 5.*) the Lines HI and GH would be equal.

PROPOSITION XXX.

A PROBLEM.

To divide a Line according to the extreme and middle Proportion.



L ET AB be the Line propos'd to be divided according to the
the

the extream and middle proportion, *i. e.* so that AB may have the same proportion to AC as AC to CB. Divide the Line ABC (*by the 11. 2.*) so, that the Rectangle contain'd under AB and CB may be equal to the Square of AC.

Demonstration.

Since the Rectangle under AB and CB is equal to the Square of AC, AB will have the same proportion to AC as AC to CB, (*by the 17.*)

The USE

• This Proposition is necessary in the Thirteenth Book of *Euclid*, for the finding the Sides of the five regular Bodies. And Friar *Lucas*, of the *Holy Sepulcher*, has compos'd a whole Book concerning the Properties of a Line divided according to the extreme and middle proportion.

PROPOSITION XXXI.

A THEOREM.

A Polygon describ'd upon the Base of a Rectangular Triangle, is equal to the two similar Polygons describ'd upon the other Sides of the same Triangle.

IF the Angle BAC of the Triangle ABC be a right Angle, the Polygon D, describ'd upon its Base BC, will be equal to the two similar Polygons F and E describ'd upon the Sides AB and AC.



Demon-

Demonstration.

The Polygons D, E, and F, are amongst themselves in the duplicate proportion of their homologous Sides BC, AC, and AB, (*by the 20.*) and if Squares were describ'd upon the same Sides, they also would amongst themselves be in the duplicate proportion of their Sides; but (*by the 47. 1.*) the Square of BC would be equal to the Squares of AC and AB: therefore the Polygon D describ'd upon the Base BC, will be equal to the similar Polygons E, and F, describ'd upon AB and AC.

The U S E.

' This Proposition is made use of to augment
' or diminish all manner of Figures, being
' more universal than the 47. 1. which yet is
' exceeding useful, in as much as almost all
' Geometry is grounded upon that Principle.

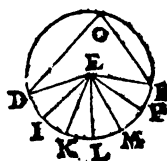
The 32 Proposition is useless.

PROPOSITION XXXIII.

A THEOREM.

In equal Circles, the Angles as well at the Center as Circumference, as also the Sectors, are in the same proportion as the Arches upon which they stand.

IF the Circles ANC and DOF are equal, the Angle ABC will have the same proportion to the Angle DEF as the Arch AC to the



Arch DF. Suppose AG, GH, and HC, to be equal Arches, and consequently the aliquot Parts of AC; and let DF be divided into as many Parts, equal to AG, as it contains; and draw the Lines EI, EK, and the rest.

Demonstration.

All the Angles, ABG, GBH, HBC, DEI, IEK, and the rest, are equal, (*by the 27. 3.*) so that AG, an aliquot Part of the Arch AC, will be contain'd in the Arch DF, as oft as the Angle ABG, an aliquot Part of the Angle ABC, is contain'd in the Angle DEF; therefore the Arch AC will have the same proportion to the Arch DF, as the Angle ABC to the Angle DEF. And because N and O are the halves of the Angles

Angles ABC and DEF, they will be in the same proportion as these : therefore the Angle N has the same proportion to the Angle O, as the Arch AC to the Arch DF.

The same holds likewise in the Sectors: for if you draw the Lines AG, GH, HC, DI, IK, and the rest, they will be equal, (*by the 29. 3.*) and each little Sector will be divided into a Triangle, and a Segment. But the Triangles will be equal, (*by the 8. 1.*) and the little Segments will also be equal, (*by the 24. 3.*) therefore the whole little Sectors will be equal; and consequently, as many aliquot Parts of the Arch AC as are contain'd in the Arch DF, so many aliquot Parts of the Sector ABC will be contain'd in the Sector DEF. Therefore the Arch has the same proportion to the Arch, as the Sector to the Sector.



THE

THE ELEVENTH BOOK
OF THE
ELEMENTS
OF
EUCLID.

THIS Book establishes the first Principles relating to solid Bodies; insomuch that it is impossible to know any thing certainly concerning the third *Species* of Quantity, without understanding what is herein taught. Upon which account the Knowledge of it is absolutely necessary to a thorough insight into the greatest Part of *Mathematical* Treatises. In the first place, the Doctrine of the Sphere, deliver'd by *Theodosius*, does suppose a perfect Knowledge of the whole. In like manner *Spherical Trigonometry*, the third Part of *Practical Geometry*, divers Propositions of *Statics* and *Geography* are built upon the Principles of Solids. The main Difficulties in *Gnomonicks*, *Conick Sections*, and the Tracts concerning the cutting of precious Stones, arising chiefly from their Eminences, and

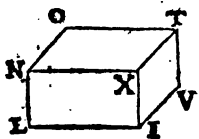
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rais'd Parts, not easily represented upon Paper, and their being contain'd under many Superficies, are render'd intelligible and easy by the previous Knowledge of the Doctrine of Solids.

I have omitted the seventh, eighth, ninth and tenth Books of the *Elements* of *Euclid*, being of little or no use in any Part of the *Mathematicks*. And I have oft wonder'd how they obtain'd a Place amongst the *Elements*, since 'tis evident *Euclid* compil'd them for no other End but to settle the Doctrine of Incommensurables; which being little better than a vain Curiosity, ought not to be receiv'd into the Books, which treat of the *first Principles* of the Science, but to make a particular Treatise by it self. The same may be said of the thirteenth Book, and those that follow it. And therefore 'tis my Opinion, that almost all Parts of the *Mathematicks* may sufficiently be understood by the help of these Eight Books of the *Elements* of *Euclid*.

DEFINITIONS.

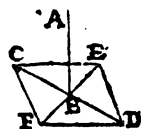


1. A Solid Body is a Quantity, that hath length, breadth, and depth, or thickness. " As the Figure DL, " whose length is NX, breadth " NO, and thickness LN.

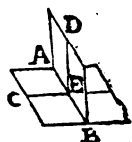
2. The

2. The Extrems or Terms of a Solid Body are Surfacies's.

3. A Line is right, or perpendicular to a Plane, when 'tis perpendicular to all the Lines, which it meets in the Plane. " As the " Line AB will be right to the " Plane CD, if it be perpendicular to the Lines " CD and FE, which being drawn upon the " Plane CD, pass by the point B, so that the " Angles ABC, ABD, ABE, and ABF, are " right Angles.

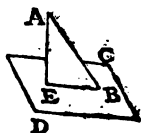


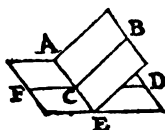
4. One Plane is perpendicular to another, when a perpendicular Line drawn upon one of them to the common Section, is also perpendicular to the other.



' We call the Line that is common to both the ' Planes, the common Section of the Planes: ' As the Line AB, which is as well in the Plane ' AC, as in the other AD. If therefore the ' Line DE, drawn on the Plane AD, perpendicular to AB, be also perpendicular to the ' Plane AC, the Plane AD will be right to the ' Plane AC.

5. If the Line AB be not perpendicular to the Plane CD, and from the Point A a Perpendicular be drawn to it, AE, and also the Line BE, the Angle ABE is the Inclination of the Line, AB to the Plane CD.





6. The Inclination of one Plane to another, is the acute Angle form'd by the two Perpendiculars drawn upon each Plane to their common Section. ' As the Inclination of the Plane AB to the Plane AD, is nothing else but the Angle BCD, form'd by the Lines BC and CD, drawn upon the two Planes, perpendicular to their common Section AE.

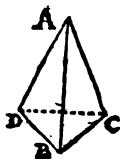
7. Planes are inclin'd after the same manner, if their Angles of Inclination are equal.

8. Planes are parallel, if being continu'd as far as you please, they still retain the same distance one from the other.

9. Solid Figures are similar which are contain'd within, or terminated by, an equal number of similar Planes; as two Cubes.

' This Definition does not agree to those Figures, whose Superficies are crooked; as the Sphere, the Cylinder, and the Cone.

10. Equal and similar Solid Figures are contain'd within, or terminated by, an equal number of equal and similar Planes. ' Infomuch, that if they were suppos'd to penetrate each other, neither of them will exceed each other, having their Sides and Angles equal.

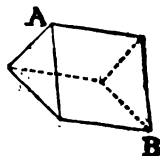


11. A solid Angle is the Concourse, or Inclination, of divers Lines, in different Planes. ' As the Concourse of the Lines AB, AC, and AD, which are in different Planes.

12. A Pyramid is a solid Figure, terminated by Triangles, whose Bases are in the same Plane. As the Figure ABCD.

13. A Parallelepipedon is a solid Figure contain'd within fix quadrilateral Planes, of which the Opposites are parallel.

14. A Prism is a solid Figure, having two parallel Plains similar and equal; and the others, Parallelograms. As in the Figure AB. Its opposite Planes may be Polygons.



15. A Sphere is a solid Figure, terminated by one only Superficies; from which divers Lines being drawn to a Point in the middle of the Figure, they will be all equal. Some define a Sphere by the motion of a Semicircle, turn'd about upon its Diameter, which remains immoveable.

16. The Axis of a Sphere, is that immoveable Line about which the Semicircle is turn'd.

17. The Center of the Sphere, is the same with that of the Semicircle, by whose Motion it is made.

18. The Diameter of a Sphere, is any Line whatsoever, passing through its Center, and terminated at the Superficies.

19. if a Line, immoveable at one of its Points, taken above the Plane of a Circle, be mov'd about the Circumference, it will describe a Cone. As if the Line AB, being fix'd at the Point A, be mov'd about the Circumference



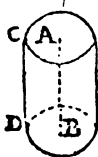
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BED,

' BED, it will describe the Cone **ABED**. The
 ' Point **A** will be its Summity or **Vertex**, and
 ' the Circle **BED** its Base.

20. The **Axis** of a Cone, is the Line drawn from the **Vertex** to the **Center** of the **Base**.

' As **AC**.



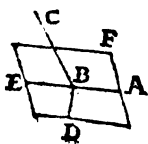
21. if a Line be mov'd about the Circumference of two parallel Circles, so that it remains always parallel to a Line drawn from the Center of one of the Circles to that of the other, *i. e.* the **Axis**, it will describe a **Cylinder**.

22. Cones are said to be right, when the **Axis** is perpendicular to the Plane of the **Base**. Also Right Cones are similar, when their **Axis's** and the **Diameters** of their **Bases** are in the same proportion. But **Inclin'd** Cones are not similar, unless they have a third Condition; that their **Axis's** be equally inclin'd to the **Planes** of their **Bases**.

PROPOSITION I.

A THEOREM.

A right Line cannot have one of its Parts upon a Plane, and the other above, or below it.



IF the Line **AB** be upon the Plane **AD**, it will not, being continu'd, either rise above, or fall below it, but all its Parts will lie upon the same. For if it be

be possible that BC can be a Part of AB continu'd, draw upon the same Plane AD the Line BD perpendicular to AB, and also BE perpendicular to BD upon the same.

Demonstration.

The Angles ABD and DBE are two right Angles; therefore (by the 14. 1.) AB and BE make but one right Line, and consequently BC is no Part of the Line AB continu'd; otherwise two right Lines CB and EB would have the same Part AB in common, which is repugnant to the 13 Axiom of the first Book.

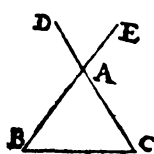
The USE.

‘ Upon this Proposition is built a Principle
 ‘ in *Gnomonicks*, by which we prove, that the
 ‘ *Shadow* of the *Style* cannot fall out of the Plane
 ‘ of a great Circle in which is the Sun For
 ‘ the Extremity of the *Style* being taken for the
 ‘ Center of the Heavens, and consequently of
 ‘ all the greater Circles; and the *Shadow* being
 ‘ always in a right Line with a Ray drawn
 ‘ from the Sun to the opacous Body, and this
 ‘ Ray being in the Plane of this great Circle,
 ‘ the *Shadow* must be so likewise.

PROPOSITION II.

A THEOREM.

Lines that cut each other, are in the same Plane, as are also all the Parts of a Triangle.



IF the two Lines BE and CD cut each other at the Point A; and a Triangle be form'd by drawing the Base BC; I say, all the Parts of the Triangle ABC are in the same Plane, and also the Lines BE and CD.

Demonstration.

It cannot be said that any Part of the Triangle ABC is in a Plane, and another Part of the same Triangle not in the same Plane, but it must be also affirm'd, that one Part of a right Line is in a Plane, and another Part of the same Line is not in the same Plane; which is contrary to *Prop. 1.* And because the Sides of the Triangle must be in the same Plane in which is the Triangle, the Line BE and CD will be also in the same Plane.

The USE.

‘ This Proposition sufficiently determines a Plane, by the concurrence of two right Lines, or by a Triangle. I have also made use of it in *Opticks*, to prove that objective parallel Lines, which meet upon a Table, ought to be represented by Lines that concur in a Point,

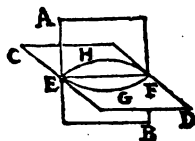
P R O.

PROPOSITION III.

A THEOREM.

The common Section of two Planes is one right Line.

IF the Planes AB and CD cut each other, their common Section EF will be one right Line. For if not, take two Points common to both Planes, as E and F; and draw a right Line from the Point E to the Point F upon the Plane AB, which suppose to be EHF. Draw likewise upon the Plane CD a right Line from the same Point E to F; and if it be not the same with the former, suppose it to be EGF.



Demonstration.

These Lines drawn upon two Planes are two different Lines, and enclose Space; which is contrary to the 12 *Axiom of the 1*. Therefore they will make but one right Line, which being in both the Planes will be their common Sections.

The USE.

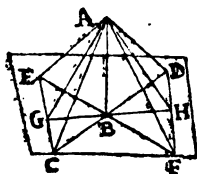
‘ This is a fundamental Proposition, suppos’d
 ‘ in divers Parts of the *Mathematicks*, though it
 ‘ be not always quoted. Particularly, it is
 ‘ taken for granted in *Gnomonicks*, when the
 ‘ Hour-Lines are represented upon *Dials*, by
 mark-

marking only the common Section of their Plane, and that of the Wall.

PROPOSITION IV.

A THEOREM.

If a Line be perpendicular to two others that cut each other, it will also be perpendicular to the Plane of the same Lines.



IF the Line AB be perpendicular to the Lines CD and EF, which cut each other at the Point B, so that the Angles ABC, ABD, ABE, and ABF, be right Angles, (which cannot conveniently be represented upon a Plate,) it will be also perpendicular to the Plane of the Lines CD and EF, *i. e.* to all the Lines that shall be drawn upon the same Plane, through the Point B, as, for example, the Line GBH. Let the Lines BC, BD, BE, and BF, be equal; and draw the Lines EC, DF, AC, AD, AE, AF, AG, and AH.

Demonstration.

The four Triangles ABC, ABD, ABE, and ABF, have each a right Angle at the Point B; and the Sides BC, BD, BE, and BF equal, with the Side AB common to all. Therefore their Bases AC, AD, AE, and AF, are equal, (*by the 4. L.*)

2. The

2. The Triangles EBC and DBF will be in all respects equal, having their Sides BC, BD, BE and BF, equal; and the Angles CBE and DBF, being opposite at the top, equal: therefore the Angles BCE, BDF, BEC, and BFD, will be equal, (*by the 4. 1.*) and also the Bases EC and DF.

3. The Triangles GBC, and DBH, having the opposite Angles CBG and EBH, equal; as also the Angles BDH, and BCG; and the Sides BC and BD; the Sides BG and BH, CG and DH, will be also equal, (*by the 26. 1.*)

4. The Triangles ACE and AFD, having the Sides AC, AD, AE, and AF, equal; and the Bases EC and DF also equal; the Angles ADF and ACE will be equal, (*by the 8. 1.*)

5. The Triangles ACG and ADH have the Sides AC and AD, CG and DH equal, with the Angles ADH and ACG; therefore their Bases AG and AH are equal.

Lastly, the Triangles ABH and ABG have all their Sides equal; therefore (*by the 8. 1.*) the Angles ABG and ABH will be equal, and the Line AB perpendicular to GH. Accordingly, the Line AB will be perpendicular to any Line drawn through the Point B upon the Plane of the Lines CD and EF, which I call being perpendicular to their Plane.

The USE.

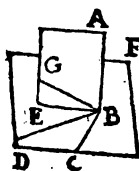
‘ This Proposition occurs very oft in the first Book of *Theodosius*: for example, to demonstrate that the *Axis* of the World is perpendicular to the Plane of the *Equinoctial*. In like
‘ man-

'manner in *Gnomonicks*, 'tis demonstrated by
 'this Proposition, that the *Equinoctial Line* in
 'Horizontal Dials is perpendicular to the *Meri-*
 'dian. Nor is it less useful in other *Mathemati-*
 'cal Treatises; as those concerning *Astrolabes*,
 'and the cutting of precious Stones.

PROPOSITION V.

A THEOREM.

If a Line be perpendicular to three others, which cut each other at the same Point, they will be all three in the same Plane.



IF the Line AB be perpendicular to three Lines BC, BD, and BE, which cut each other at the same Point B; the Lines BC, BD, and BE are in the same Plane. Suppose the Plane AE to be that of the Lines AB and BE, and CF that of the Lines BC and BD. If BE be the common Section of both the Planes, it will be in the Plane of the Lines BC and BD, as was asserted: but if BE be not, let BG be their common Section.

Demonstration.

AB is perpendicular to the Lines BC and BD, therefore it is perpendicular to their Plane CF, (*by the 4.*) and (*by Defn. 3.*) AB will be perpendicular to BG. But it is also suppos'd perpen-
dicular

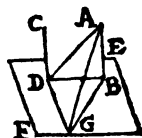
dicular to BE ; therefore the Angles ABE and ABG are right Angles, and consequently equal, though one be but part of the other. Therefore the two Planes can have no other common Section but BE. BE is therefore in the Plane CF.

PROPOSITION VI.

A THEOREM.

Lines that are perpendicular to the same Plane, are parallel.

IF the Lines AB and CD be perpendicular to the same Plane EF, they will be parallel. 'Tis evident, that the internal Angles ABD and BDC are right Angles ; but that is not enough ; it remains to be prov'd, that AB and CD are in the same Plane. Draw DG perpendicular to BD, and equal to AB ; draw also the Lines BG, AG, and AD.



Demonstration.

The Triangles ABD and BDG have the Sides AB and DG equal, and BD common to both ; and the Angles ABD and BDG are right Angles, therefore their Bases AD and BG are equal, (*by the 4. 1.*) Further, the Triangles ABG and ADG have all their Sides equal : therefore the Angles ABG and ADG are equal ; and ABG being a right Angle, because AB is perpendicular to the Plane

Plane, ADG is also a right Angle. Therefore the Line DG is perpendicular to three Lines CD, DA, and DB, which consequently are in the same Plane, (*by the 5.*) But the Line AB is in the Plane of the Lines AD and DB, (*by the 2.*) therefore AB and CD are in the same Plane.

Coroll. Two parallel Lines are in the same Plane.

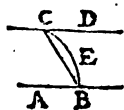
The U S E.

‘ By this Proposition we demonstrate, that
 ‘ the Hour-lines, in all Planes that are parallel
 ‘ to the *Axis* of the World, as the Polars, Meri-
 ‘ dional, and others, are parallel among them-
 ‘ selves.

PROPOSITION VI.

A THEOREM.

A Line drawn from one Parallel to another, is in the same Plane with them.



THE Line CB, being drawn from the Point B of the Line AB to the Point C of the Parallel CD, is (I say) in the Plane with the Lines AB and CD.

Demonstration.

The Parallels AB and CD are in the same Plane: in which if you draw a right Line from the

the Point B to the Point C, it will be the same with CB; otherwise two right Lines would enclose Space, contrary to the 12. Axiom of the 1.

PROPOSITION. VIII.

A THEOREM.

If one of two parallel Lines be perpendicular to a Plane, the other will be so also.

IF of the two parallel Lines AB and CD, [see Fig. Prop. 6.] the one AB be perpendicular to the Plane EF, the other CD will be so also. Draw the Line DB: since the Angle ABD is a right Angle, and AB and CD are suppos'd to be Parallels, the Angle CDB will be a right Angle, (by the 29. 1.) therefore if I can prove, that the Angle CDG is also a right Angle, it will follow (by the 4.) that CD is perpendicular to the Plane EF. Make a right Angle BDG, and take DG equal to AB; then draw the Lines EG and AG.

Demonstration.

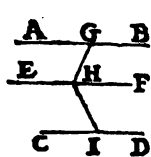
The Triangles ABD and BLD have the Sides AB and DG equal, with the Side BD common to both; and the Angles ABD and BDG are right Angles; therefore (by the 4. 1.) their Bases AD and BG are equal. The Triangles ADG and ABG have all their Sides equal; therefore (by the 4. 1.)

the 8. 1.) the Angles ADG and ABG are equal : but the latter is a right Angle, because AB is suppos'd to be perpendicular to the Plane EF, therefore the Angle ADG is a right Angle ; and the Line DG being perpendicular to the Lines DB and DA, will be perpendicular to the Plane of the Lines DB and DA, which is the same, in which are the Parallels AB and CD. Therefore the Angle GDC is a right Angle, (*by Definit. 3.*)

PROPOSITION IX.

A THEOREM.

Lines that are parallel to a third, are also parallel among themselves, though not all in the same Plane.



IF the Lines AB and CD are parallel to the Line EF, they will be parallel to each other, though all the three Lines be not in the same Plane. Upon the Plane of the Lines AB and EF draw the Line HG perpendicular to AB ; which will be also perpendicular to EF, (*by the Lemma after the 26. 1.*) In like manner, upon the Plane of the Lines EF and CD draw the Line HI perpendicular to EF and CD.

Demon-

Demonstration.

The Line EH being perpendicular to the Lines GH and HI, is so also to the Planes of the Lines HG and HI, (*by the 4.*) therefore (*by the 8.*) the Lines HG and CI are perpendicular to the Plane of the Lines AH and FI, and (*by the 6.*) parallel to each other.

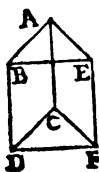
The USE.

‘ This Proposition is frequently us’d in *Perspectives*, to determine the representation of
 ‘ parallel Lines upon a Table; as also in the
 ‘ cutting of precious Stones, to prove the Sides
 ‘ of the Pannels to be parallel among them-
 ‘ selves, because they are so to a Line in a dif-
 ‘ ferent Plane. In *Gnomonicks* likewise we are
 ‘ sometimes oblig’d to prove, that the Vertical
 ‘ Circles ought to be describ’d on Walls by per-
 ‘ pendicular Lines; because the Lines, that are
 ‘ the common Sections of them and the Walls,
 ‘ are parallel to a Line drawn from the *Zenith*
 ‘ to the *Nadir*.

PROPOSITION X.

A THEOREM.

If two Lines, which concur, are parallel to two others concurring, in a different Plane, they will make equal Angles.



IF the Lines AB and CD, AE and CF be parallel, though they be not all four upon the same Plane, yet the Angles BAE and DCF will be equal. Let the Lines AB and CD, AE and CF be equal, and draw the Lines BE, DF, AC, BD, and EF.

Demonstration.

The Lines AB and CD are suppos'd to be both parallel, and equal, therefore (*by the 33. 1.*) the Lines AC and BD are parallel and equal, as also AC and EF; and (*by the preceeding*) BD and EF are parallel, and equal, and consequently (*by the 33. 1.*) BE and DF will be also parallel and equal. Therefore the Triangles BAE and DCF have all their Sides equal; and (*by the 8. 1.*) the Angles BAE and DCF will be equal.

Coroll. Many the like Propositions might be made, which would not be altogether unuseful; as for example, if upon a parallel Plane the Line CD be drawn parallel to the Line AB and the

the Angles BAE and DCF be equal, the Lines AE and CF will be parallel.

The U S E.

‘ By this Proposition we demonstrate, that
‘ the Angles made by the Planes of the Hour-
‘ circles with a Plane parallel to the *Equator*,
‘ are equal to the Angles made by them with
‘ the Plane of the *Equator*.

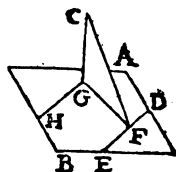
PROPOSITION XI.

A PROBLEM.

*To draw a perpendicular to a Plane from a Point
given out of the Plane*

IF you desire to draw a perpendicular from the Point C to the Plane AB, draw the Line ED at pleasure, and CF perpendicular to it, (*by the 12. 1.*)

And again (*by the 11. 1.*) upon the Plane AB draw FG perpendicular to ED, and CG perpendicular to FG. I say, CG will be perpendicular to the Plane AB. Draw GH parallel to FE.



Demonstration.

The Line EF being perpendicular to the Lines CF and FG, will be perpendicular to the Plane CFG, (*by the 4.*) and HG being parallel to EF, will be also perpendicular to the same Plane, (*by the 8.*) And because CG is perpendi-

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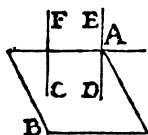
cular

cular to the Lines GF and GH, it will be perpendicular to the Plane AB, (*by the 4.*)

PROPOSITION XII.

A PROBLEM.

To draw a Perpendicular to a Plane through a Point of the same Plane.

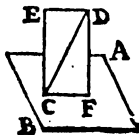


TO draw a perpendicular to the Plane AB through the Point C. Draw from the Point E, taken at pleasure out of the Plane, the Line ED perpendicular to the same Plane, (*by the 11.*) Draw also (*by the 31. 1.*) CF parallel to DE. CF will be perpendicular to the Plane AB, (*by the 8.*)

PROPOSITION XIII.

A THEOREM.

Two Lines perpendicular to a Plane cannot be drawn through the same Point.



IF the two Lines CE and CD drawn through the same Point C, were perpendicular to the Plane AB; and CF the common Section of the Planes of those Lines,

Lines, with the Plane AB; the Angles ECF and DCF would be both right Angles, which is impossible.

I add, that two Perpendiculars DC and EF, to the Plane AF cannot be drawn from the same Point D : for having drawn the Line CF, there would be two right Angles, DCF and DFC, in the same Triangle, *contrary to the 32. 1.*

The *USE*

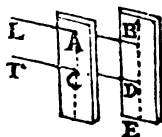
This Proposition is necessary to show, that a Perpendicular to a Plane was sufficiently describ'd, in as much as but one such can be drawn through the same Point.

PROPOSITION XIV.

A THEOREM.

Planes, to which the same Line is perpendicular, are parallel.

IF the Line AB be perpendicular to the Planes AC and BD, they will be parallel, *i. e.* they will in all places be equally distant from each other. Draw the Line DC parallel to AB, (*by the 31. 1.*) and join the Lines BD and AG.



Demonstration.

AB is suppos'd to be perpendicular to the Planes AC and BD, therefore the Line CD, which

which is parallel to it, will be also perpendicular to them, (*by the 8.*) and consequently the Angles B and D, A and C, will be right Angles; and (*by the 28. 1.*) the Lines AC and BD will be Parallels, and the Figure ABDC a Parallelogram. Therefore the Lines AB and CD are equal, (*by the 34. 1.*) i. e. the Planes in the Points A and C, B and D, are equally distant. Accordingly the Line CD may be drawn through any other Point whatsoever; therefore the Planes AC and BD are equally distant, in all Places, the one from the other.

The U S E.

‘ *Theodosius* demonstrates the Circles, that
 ‘ have the same Poles, as the *Equator*, and
 ‘ the two *Tropicks*, to be parallel, because the
 ‘ *Axis* of the World is perpendicular to their
 ‘ Planes.

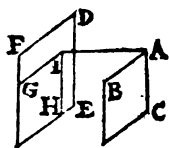
PRO-

PROPOSITION. XV.

A THEOREM.

If two Lines, meeting at a Point, be parallel to two Lines of another Plane, the Planes of those Lines will be parallel.

IF the Lines AB and AC be parallel to the Lines DE and DF, which are in another Plane, the Planes BC and FE are parallel. Draw AI perpendicular to the Plane BC, (*by the 11.*) and GI and IH, parallel to FD and DE: they will be also parallel to the Lines AB and AC, (*by the 9.*)



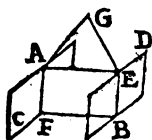
Demonstration.

The Lines AB and GI are parallel, and the Angle IAB is a right Angle, AI being perpendicular to the Plane BC: Therefore (*by the 29. 1.*) the Angle AIG is a right Angle, as also the Angle AIH. Therefore (*by the 4.*) the Line AI is perpendicular to the Plane GH; and being also perpendicular to the Plane BC, the Planes BC and GH, or FE will be parallel, (*by the 14.*)

PROPOSITION XVI.

A THEOREM.

If a Plane cut two others which are parallel, their common Sections will together with them be parallel.



IF the Plane AB cut two other parallel Planes, AC and BD; I say their common Sections AF and BE will be parallel. For if not, being continu'd, they would at length concur, *e. g.* at the Point G.

Demonstration.

The Lines AF and BE are upon the Planes AC and BD; and therefore (*by the 1.*) can never be either above, or below them; therefore if they concur at the Point G, the Planes must do so likewise, and consequently they would not be parallel, which is contrary to what was suppos'd.

The USE.

By this Proposition we demonstrate, in the Treatise of Conick and Cyndrick Sections, that if the Cone or Cylinder be cut by a Plane parallel to its Base, the Sections are Circular. By the same we describe *Astrolabes*; and prove in *Gnomonicks*, that the Angles, which the Hour-circles make with a Plane parallel to a great Circle, are equal to those which they make

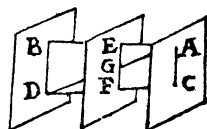
‘ make in the Circle it self; and again in *Perspectives*, that the Images of the objective Lines perpendicular to the Table, concur at the Point of Sight.

PROPOSITION XVII.

A THEOREM.

Two Lines are divided proportionally by parallel Planes.

IF the Lines AB and DC be divided by parallel Planes, I say, AE will have the same proportion to EB as CF to FD. Draw the Line AD, passing through the Plane EF at the Point G: Draw also AC, BD, FG, and GE.



Demonstration.

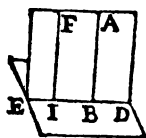
The Plane of the Triangle ABD cuts the three Planes, therefore (*by the 16.*) the Sections BD and EG are parallel; and (*by the 2. 6.*) AE has the same proportion to EB, as AG to GD. In like manner the Plane of the Triangle ADC cuts the Planes EF and AC, therefore the Sections AC and FG are parallel; and FC has the same proportion to FD as AG to GD, *i. e.* as AE to EB.

PRO-

PROPOSITION XVIII.

A THEOREM.

If a Line be perpendicular to a Plane, all the Planes, in which that Line is found, are perpendicular to the same Plane.



IF the Line AB be perpendicular to the Plane ED, all the Planes in which it is found will be perpendicular to the Plane ED. Suppose AB to be in the Plane AE, having for a common Section with the Plane ED the Line BE ; to which draw a Perpendicular FI.

Demonstration.

The Angles ABI and BIF are right Angles, therefore the Lines AB and FI are parallel ; and (*by the 8.*) FI will be perpendicular to the Plane ED. Therefore the Plane AE will be perpendicular to the Plane ED, (*by Defn 4.*)

‘ The same may be prov’d of the Plane AD.

The USE.

‘ The first Proposition in *Gnomonicks*, which
 ‘ may pass for a fundamental one, is built upon
 ‘ this Proposition ; which is also frequently
 ‘ made use of, in *Spherical Trigonometry*, in *Per-*
 ‘ *spectives*, and generally in all those Treatises
 ‘ which are oblig’d to consider divers Planes.

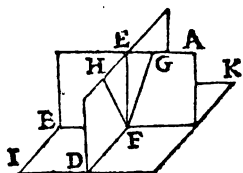
P R O-

PROPOSITION XIX.

A THEOREM.

If two Planes cutting each other be perpendicular to another, their common Section will be perpendicular to the same.

IF the Planes AB and ED, which cut each other, be perpendicular to the Plane IK; their common Section EF is also perpendicular to the Plane IK.



Demonstration.

IF EF be not perpendicular to the Plane IK, upon the Plane AB draw the Line GF perpendicular to the common Section BF: and the Plane AB being perpendicular to the Plane IK, the Line GF will be perpendicular to the same Plane. Draw likewise FH perpendicular to the common Section DF; it will be also perpendicular to the Plane IK. We shall have therefore two Perpendiculars to the same Plane drawn through the same Point F, (*contrary to the 13. Propos.*) it must therefore be granted that EF is perpendicular to the Plane IK.

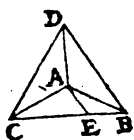
The

‘ By this Proposition we demonstrate, that
 ‘ the Circle which passes through the Poles of
 ‘ the World and the *Zenith*, is the *Meridian*,
 ‘ and cuts all the diurnal Arches into two equal
 ‘ Parts; and that the Stars spend as much time
 ‘ in their Motions from their Risings to this
 ‘ Circle, as from this Circle to their Settings.

PROPOSITION XX.

A THEOREM.

If three plain Angles make one solid one, any two of them ought to be greater than the third.



IF the Angles BAC, BAD, and CAD, make the solid Angle A, and the Angle BAC be the greatest Angle; the two others, taken together, are greater than BAC. Suppose the Angle CAE to be equal to the Angle CAD, and the Lines AD and AE to be equal; and draw the Lines CEB, CD, and BD.

Demonstration.

The Triangles CAE and CAD have the Sides AD and AE equal, and the Side AC common to both, and the Angles CAD and CAE equal; therefore (*by the 4. 1.*) their Bases CD and CE are equal. But the Side CD and DB are greater than the Side CB alone, (*by the 2c. 1.*) therefore taking

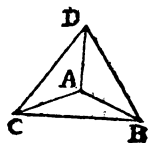
taking away the equal Lines CD and CE, the Line BD will be greater than BE. Further, the Triangles BAE and BAD have the Sides AE and AD equal, and the Side BA common, and the Base BD greater than the Base BE: therefore (*by the 18. 1.*) the Angle BAD is greater than the Angle BAE; adding therefore the equal Angles CAD and CAE, the Angles BAD and CAD will be greater than the Angles CAE and BAE, *i. e.* the Angle BAC.

PROPOSITION XXI.

A THEOREM.

All the plain Angles, that make one solid Angle, are less than four right Angles.

IF the plain Angles BAC, BAD and CAD, make the solid Angle A, they will be less than four right Angles. Draw the Lines BC, BD, and CD, and you will have a Pyramid, whose Base is the Triangles BCD.



Demomstration.

The solid Angle at the Point B, has the Angles ABC and ABD greater than that of the Base alone CBD. In like manner ACB and ACD are greater than ACD alone, and the Angles ADC and ADB are greater than CDB alone. But all the Angles of the Base are equal to two right

right Angles, therefore the Angles ABC, ABD, ACB, ACD, ADC, and ADB, are greater than two right Angles. And because all the Angles of the Three Triangles BAC, BAD, and CAD, are equal to six right Angles; taking away more than two right Angles, there will remain less than four, for the Angles made at the Point A. But if the solid Angle A consist of more than three plain Angles, so that the Base of the Pyramid be a Polygon, it may be divided into Triangles, and the Computation being made, you will find, that all the plain Angles, which make up the solid one, are always less than four right Angles.

The U S E.

‘ These two Propositions show when many
 ‘ plain Angles may make up one solid one,
 ‘ which is often necessary in the Treatises of
 ‘ cutting of Stones, and in the following Pro-
 ‘ positions.

The 22. and 23. Propositions are of no use.

PRO-

PROPOSITION XXIV.

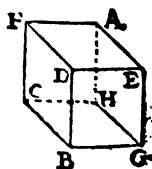
A THEOREM.

If a solid Body be terminated by parallel Planes, the opposite Sides will be similar and equal Parallelograms.

IF the Solid AB be terminated by parallel Planes, the opposite Superficies's will be similar and equal Parallelograms.

Demonstration.

The parallel Planes AC and BE are cut by the Plane FE: therefore their common Sections are parallel, (*by the 16.*) and so likewise DF and AE: therefore AD will be a Parallelogram. After the same manner I may demonstrate, that AG, FB, CG, and the rest are Parallelograms. I add, that the opposite Parallelograms, *e. g.* AG and FB, are similar and equal. The Lines AE and EG are parallel to the Lines FD and DB: therefore the Angles AEG and FDB are equal, (*by the 10.*) Accordingly I may demonstrate all the Sides and all the Angles of the opposite Parallelograms to be equal, therefore the Parallelograms are similar and equal.

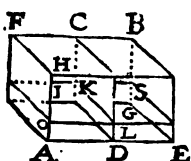


PRO-

PROPOSITION XXV.

A THEOREM.

If a Parallelepipedon be divided by a Plane parallel to one of its Planes, the two solid Bodies which arise by that Division, will have the same Proportion as their Bases.



IF the Parallelepipedon AB be divided by the Plane CD, which is parallel to the Planes AF and BE, the Solid AC will have the same proportion to BD as the Base AI to the Base DG. Suppose the Line AH, which shows the height of the Figure to be divided into as many equal Parts as you please; for example, ten thousand, which we may take as Indivisibles, *i e.* without reflecting upon the possibility of their being further subdivided. Suppose also so many Superficies's parallel to the Base AI, as there are Parts in the Line AH; I have describ'd only on OS, so that the Solid AB be compounded of all those Superficies's of the same thickness, as a Ream of Paper is compounded of all its Sheets and Quires laid one upon another. 'Tis evident that so the Solid AC will be compounded of ten thousand Superficies's equal to the Base AI, (*by the preceeding,*) and the Solid

Solid DB will contain ten thousand Superficies's equal to the Base DG.

Demonstration.

Every Superficies of the Solid AC has the same proportion to any of the Superficies's of the Solid DB as the Base AI to the Base DG; because they are every one of them equal to their Bases: therefore (*by the 12. 5.*) all the Superficies's of the Solid AC taken together, will have the same proportion to all those of the Solid DB, as the Base AI to the Base DG. But all the Superficies's of the Solid AC make up the Solid AC, which has no other Parts but those Superficies's: and all the Superficies's of the Solid DB are nothing else but the Solid DB; therefore the Solid AC has the same proportion to the Solid DB, as the Base AI to the Base DG.

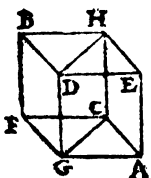
The USE

' This is *Cavalierius's* Demonstration; which
' is very clear, provided it be us'd as it ought,
' and that the Line, by which is measur'd the
' thickness of the Superficies's, be taken in the
' same respect in both the Terms. I shall make
' use of it hereafter, to render some intricate
' and perplex'd Demonstrations more facil and
' clear.

PROPOSITION XXVI.

A THEOREM.

A Parallelepipedon is divided into two equal Parts by the diagonal Plane.



S Suppose the Parallelepipedon AB to be divided by the Plane CD, drawn from one Angle to another : I say it will be divided into two equal Parts. Divide the Line AE into as many Parts as you please, and draw so many Planes parallel to the Base AF ; each of those Planes is a Parallelogram equal to the Base AF, (*by the 24.*)

Demonstration.

All the Parallelograms that can be drawn parallel to the Base AF, are divided into two equal Parts by the Plane CD : for the Triangles which are form'd on both Sides the Plane CD, have their Base common, in each equal to CD, and their Sides equal, being those of a Parallelogram. But tis evident, that the Parallelepipedon is nothing else but those Parallelograms, which are each divided into two equal Triangles : therefore the Parallelepipedon is divided into two equal Parts by the Plane CD.

The 27. and 28. Propositions are of no use according to this way of demonstrating.

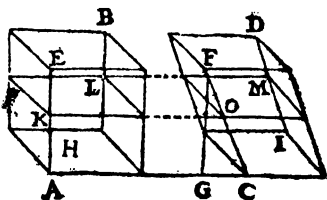
P R O-

PROPOS. XXIX, XXX, XXXI.

A THEOREM.

Parallelepipedons of the same height, having the same or equal Bases, are Equal.

IF the Parallelepipedons AB and CD be of the same height according to the Perpendiculars AE and FG, and have the same or equal



Bases AH and CI, they will be equal. Suppose the two Bases be set upon the same Plane; since their Perpendiculars are equal, the Bases EB and FD will be in the same Plane, which will be parallel the Plane of the Base AH and CI. Suppose then the Line AE or FG to be divided into as many equal Parts as you please, e. g. ten thousand, and according to them so many Superficies's, or Planes drawn of the same thickness: I have describ'd only one for all, as K or M. Each Superficies will form in these Solids a parallel Plane, similar and equal to the Base, (by the 24.) as KL, OM; and there will be as many in one Solid as in the other; because their thickness, which I take perpendicularly according to their *respective* Heights, is equal.

X 2

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Demonstration.

The Base AH has the same proportion to the Base CI, as each Plane KL to OM. But they being equal in number in both, all the Antecedents (*by the 2. 5.*) will have the same proportion to all the Consequents, *i. e.* the whole Solid AB to the whole Solid CD, as the Base AH to the Base CI. But 'tis suppos'd that the Bases are equal, therefore the Solids are equal.

Coroll. To find the Solidity of a Parallelepipedon, 'tis usual to multiply the Base by the height taken perpendicularly, because that Perpendicular shows how many Superficies's equal to the Base are contain'd in it. As for example, if I take a Foot for my Indivisible Measure, *i. e.* which I will not afterwards subdivide; if the Base contain twelve Feet square, and the perpendicular height ten, I shall have an hundred and twenty Cubick Feet for the solidity of the Body AB. For the height containing ten Feet, I may make ten Parallelograms equal to the Base, having each a Foot in thickness; but the Base with one Foot in thickness makes twelve Cubick Feet: the whole therefore will make an hundred and twenty, if the height contain ten Feet.

P R O-

PROPOSITION XXXII.

A THEOREM.

Parallelepipedons of the same height are in the same Proportion as their Bases.

I Have prov'd this Proposition in the preceding, demonstrating, that the Parallelepipedon AB has the same proportion to the Parallelepipedon CD, as the Base AH to the Base CI. (See Fig. preced.)

Coroll. Parallelepipedons that have equal Bases, are in the same proportion as their Heights. As the Parallelepipedons AB and AL, whose perpendicular Heights are AK and AE. For if you divide the Height AK into as many aliquot Parts as you please, and AE into as many as it contains equal to the former, and draw, according to each Part, Planes parallel to the Base; as many as AE contains of the aliquot Parts of AK, so many will the Solid AB contain of the Superficies's equal to the Base, which are the aliquot Parts of the Solid AL; therefore (by Defin. 5. 5.) the Solid AB will have the same proportion to the Solid AL, as the Height AE to the Height AK.

The USE.

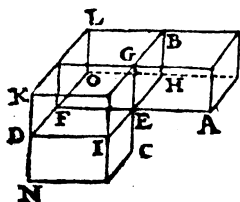
' The three preceding Propositions contain
' almost all the ways of measuring Parallelepipedons, and may be esteem'd as first Principles

'ples for that purpose. 'Tis after the same
'manner also that we take the Dimensions of
'the Solidity of Walls, by multiplying their
'Bases by their Heights.

PROPOSITION XXXIII.

A THEOREM.

Similar Parallelepipeds are in the triplicate Proportion of their homologous Sides.



IF the Parallelepipeds AB and CD be similar, *i. e.* if all the Planes of one be like those of the other; and all their Angles equal, so that they may be plac'd in a right Line, *i. e.* that AE and EF, HE and EI, GE and EC, may make right Lines; and AE has the same proportion to EF, as HE to EI, and as GE to EC: I say, that here are four Solids in continual proportion according to the proportion of the Side EA to that, which is homologous to it, EF or DI.

Demonstration.

The Parallelepipedon AB has the same proportion to EL of the same Height, as the Base AH to the Base EO, (*by the 32.*) But the Base AH has the same proportion to the Base EO, as AE to EF, (*by the 1. 6.*) In like manner, the pro-

proportion of the Solid EL to the Solid EK, is the same with that of the Base EO to the Base ED, *i. e.* that of HE to EI. And lastly, the Solid EK has the same proportion to the Solid EN, as the Height GE to the Height EC, (*by the Coroll. of the 32.*) or (taking the Line EF for their common Height) as the Base GI to the Base CI, *i. e.* as GE to EC. But the proportion of AE to EF, of HE to EI, and of GE to EC, was suppos'd to be the same; and consequently, the Solid AB has the same proportion to EL as EL to EK, and as EK to CD. Therefore (*by Defn. 11. 5.*) the proportion of AB to CD will be the triplicate proportion of that of AB to EL, or of AE to its homologous Side EF.

Coroll. If four Lines be in continual proportion, the Parallelepipedon describ'd upon the first, has the same proportion to another similar Parallelepipedon describ'd upon the second, as the first to the fourth; for the proportion of the first to the fourth, is the triplicate proportion of that of the first to the second.

The U S E.

' You may perceive by this Proposition that
' that famous Problem of the Duplication of the
' Cube, propos'd by the Oracle, consists in find-
' ing two middle Terms in continual proportion.
' For if you make the side of the first Cube the
' first Term, and the double of that the fourth;
' having found two middle Proportionals, the
' Cube describ'd upon the first Line will have
' the same proportion to that describ'd upon

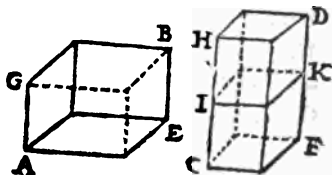
the second, as the first Line to the fourth,
i. e. as one to two. By this Proposition also
may be corrected their Error, who fancy similar Solids to have the same proportion as their Sides; as if a Cube of one foot in length was the half of a Cube two Foot long; when indeed it is but the eighth Part thereof. This is likewise the Foundation of the Rule concerning the size of the Bores of Cannons: and is applicable not only to Bullets, but to all sorts of similar Bodies. For example; should a Man about to build a Navy, and resolving to retain the same proportion in all his Vessels, reason thus with himself; if a Ship of an hundred Tun require fifty Foot in Keel: another of two hundred Tuns ought to have an hundred Foot in Keel; he would be guilty of a great Mistake: for instead of making a Vessel twice as large as the former, he would make one eight times so much. He ought to assign to the second Vessel somewhat less than sixty three Feet.

PROPOSITION XXXIV.

A THEOREM.

Equal Parallelepipedons have their Bases and Heights reciprocal, and those that have their Bases and Heights reciprocal are equal.

IF the Parallelepipedons AB and CD be equal, their Bases and Heights will be reciprocal, *i. e.* the Base AE



will have the same proportion to the Base CF, as the Height CH to the Height AG. Having make CI equal to AG, draw the Plane IK parallel to the Base CF.

Demonstration.

The Parallelepipedon AB has the same Proportion to CK, being of the same height, as the Base AE to the Base CF, (*by the 32.*) But as AB to CK, so is CD to the same CK, because AB and CD are equal; and as CD to CK, which have both the same Base, so is the Height GH to the Height CI, (*by the Coroll. of the 32.*) therefore as the Base AE to the Base CF, so is the Height CH to the Height CI or AG.

I add, that if the Base AE has the same proportion to the Base CF as the Height CH to the Height AG, the Solids AB and CD will be equal.

Demon-

Demonstration.

AB has the same proportion to CK, being of the same Height, as the Base AE to the Base CF, (*by the 32.*) Also the Height CH has the same proportion to the Height CI or AG, as CD to CK: but we suppose that AE has the same proportion to CF, as CH to CI or AG, therefore the Solid AB has the same proportion to the Solid CK, as the Solid CD to the same CK, and consequently the Solids AB and CD are equal, (*by the 9. 5.*)

The U S E.

‘ This Reciprocation of the Bases and
 ‘ Heights makes the Solid very easy to be mea-
 ‘ sur’d. And the Proposition seems to bear
 ‘ some Analogy to the 14. *Prop. of the 6.* which
 ‘ asserts, That equiangular and equal Paralle-
 ‘ lograms have their Sides reciprocal; and the
 ‘ Practice of the Rule of Three may be demon-
 ‘ strated from both.

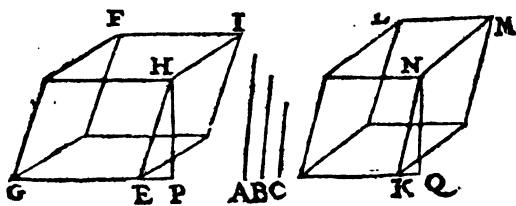
The 35. Prop. may be omitted.

PRO-

PROPOSITION XXXVI.

A THEOREM.

If three Lines be in continual proportion, a Parallelepipedon made of those three Lines is equal to an equiangular Parallelepipedon, which has all its Sides equal to the middle Line.



IF the Lines A, B, C, be in continual proportion, the Parallelepipedon EF made of those three Lines, the Side FI being equal to the Line A, HE equal to B, and HI equal to C, is equal to the equiangular Parallelepipedon KL, whose Sides LM, MN, and KN, are each of them equal to the Line B. From the Points H and N draw the Lines HP and NQ perpendicular to the Planes of the Bases; which Lines will be equal, because the solid Angles E and K are suppos'd equal, so that if they could penetrate, neither would exceed the other, and the Lines EH and KN are also suppos'd equal. Therefore the Heights HP and NQ are equal.

Demon-

Demonstration.

There is the same proportion of A to B, or of FI to LM, as of B to C, or LM to HI: therefore the Parallelogram FH contain'd under FI and IH, is equal to the Parallelogram LN contain'd under LM and MN, both equal to B, (*by the 16. 6.*) therefore the Bases are equal. But the Heights HP, and NQ are also equal; therefore (*by the 31.*) the Parallelepipedons are equal.

PROPOSITION XXXVII.

A THEOREM.

If four Lines be proportional, the Parallelepipedons describ'd upon those Lines are proportional: and if the similar Parallelepipedons be proportional, their homologous Sides will be also proportional.

$\frac{A}{C} = \frac{B}{D}$ **I** F the Line A has the same proportion to B as C to D, the similar Parallelepipedons, whose homologous Sides are the Lines A, B, C, D, will be in the same proportion.

Demonstration.

The Parallelepipedon A is in the triplicate proportion to the Parallelepipedon B, of that of the Line A to the Line B, or that of the
Line

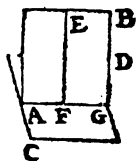
Line C to the Line D. But the Parallelepipedon C to the Parallelepipedon D is also in the triplicate proportion of that of the Line C to the Line D, (*by the 33.*) Therefore the Parallelepipedon A has the same proportion to the Parallelepipedon B, as the Parallelepipedon C to the Parallelepipedon D.

PROPOSITION XXXVIII.

A THEOREM.

If two Planes be perpendicular to each other, a Perpendicular drawn from a Point in one of the Planes to the other will fall upon the common Section.

IF the Planes AB and CD being perpendicular to each other, you draw from the Point E in the Plane AB a Line perpendicular to the Plane CD, it will fall upon the common Section of the Planes. Draw EF perpendicular to the common Section AG.



Demonstration.

The Line EF, perpendicular to AG the common Section of the Planes, which are suppos'd to be perpendicular, will be perpendicular to the Plane CD, (*by Defn. 3.*) and because two Lines cannot be drawn from the Point

E

E perpendicular to the Plane CD, (*by the 13.*) every Perpendicular will fall upon the common Section AG.

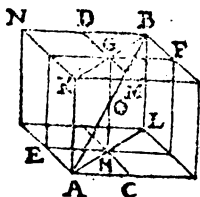
The USE.

‘ This Proposition ought to have follow’d next after the 17th, because it respects Solids in general. ’Tis of use to us in the Treatise of *Astrolabes*, to prove that in the *Analemma* all the Circles, perpendicular to the *Meridian*, ought to be markt by right Lines.

PROPOSITION XXXIX.

A THEOREM.

If in a Parallelepipedon be drawn two Planes, which divide the opposite Sides into two equal Parts, their common Section and the Diameter will also divide each other into two equal Parts.



S Uppose the opposite Sides of the Parallelepipedon AB to be divided into two equal Parts by the Planes CD and EF, their common Section GH and the Diameter BA will equally divide each other at the Point O. Draw the Lines BG, GK, AH and HL. I shall prove first, that the two first of these, BG and GK, (and so likewise AH and HL,)

HL,) make but one right Line. For the Triangles DGB and KMG have their Sides DB and KM equal, because they are the Halves of equal Sides; as also GD and GM. Further, DB and KM being parallel, the alternate Angles BDG and GMK will be equal, (*by the 29. 1.*) and therefore (*by the 4. 1.*) the Triangle DBG and KGM will be equal in all respects, and consequently the Angles BGD and KGM; and (*by the Coroll. of the 15. 1.*) BG and GK make but one right Line, as also LH and HA: therefore ALBK is one Plane, in which are found both the Diameter AB, and the common Section of the Planes GH. The Plane ALBK cutting the parallel Planes AN and CD, their common Sections GH and AK will be parallel: and (*by the 2. 6.*) BG will have the same proportion to GK, as BO to OA; and therefore (*by the 18. 5.*) as BK to GK, so BA to BO; and (*by the 4. 6.*) so GH or AK to OG. But BK is double to BG, therefore AK, equal to GH, double to GO. Therefore the Lines GH, and AB divide each other equally at the Point O.

Coroll. 1. All the Diameters are divided at the Point O.

Coroll. 2. Here we may add some Corollaries, which depend upon divers Propositions. As for example, that triangular Prisms of the same height are in the same proportion as their Bases: for the Parallelepipedons, of which they are the halves, are (*by the 32.*) in the same proportion as their Bases: therefore the halves

of their Bases, and the halves of the Parallelepipedons, *i. e.* the Prisms, will be in the same Proportion.

Coroll. 3. Polygon Prisms of the same height are also in the same proportion as their Bases, because they may be resolv'd into triangular ones, each of which will have the same proportion as their Bases.

Coroll. 4. The rest of the Propositions concerning Parallelepipedons are also applicable to Prisms: as for example, that equal Prisms have their Heights and Bases reciprocal; and that similar Prisms are in the triplicate proportion of that of their homologous Sides.

The U S E.

‘ This Proposition may help us to find out
‘ the Center of Gravity in Parallelepipedons;
‘ and to demonstrate some other Propositions
‘ in the thirteenth and fourteenth Books of
‘ *Euclid*.

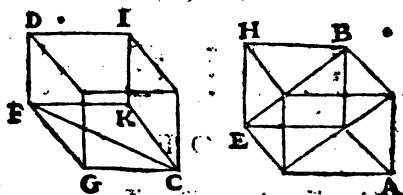
PROPOSITION XL.

A THEOREM.

A Prism, that has a Parallelogram for its Base double to the triangular Base of another Prism, and of the same height, is equal to it.

LET ABE and CDG be two triangular Prisms, of the same height; and the Base
of

of one the Parallelogram AE, double to the Triangle FGC, the Base of the other Prism: I say these Prisms are equal. Suppose the Parallelepipedon AH and GI were compleated.



Demonstration.

'Tis suppos'd that the Base AE is double to the Triangle FGC, but the Parallelogram GK is double to the same Triangle, (*by the 34. 1.*) therefore the Parallelograms AE and GK are equal; and consequently, the Parallelepipedons AH and GI, having the same Bases and the same Heights, are equal; and therefore the Prisms, that are the Halves, (*by the 26.*) will be likewise equal.

THE TWELFTH BOOK
OF THE
ELEMENTS
OF
EUCLID.

‘ **E** *Uclid*, after having in the preceeding
 ‘ Books deliver’d the general Principles
 ‘ of solid Bodies, and explain’d the
 ‘ manner of Measuring the most regular of
 ‘ them, that is, such as are terminated by plain
 ‘ Superficies’s; treats in this of such Bodies
 ‘ as are contain’d in Superficies’s that are crook-
 ‘ ed, as the Cylinder, Cone, and Sphere:
 ‘ comparing one with another, and giving
 ‘ Rules, relating both to their Solidity, and
 ‘ the manner of taking their Dimensions. The
 ‘ Book is of exceeding great use, because in it
 ‘ we find the Principles, upon which the most
 ‘ learned *Mathematicians* have built so many
 ‘ famous Demonstrations concerning the Cylin-
 ‘ der, the Cone, and the Sphere.

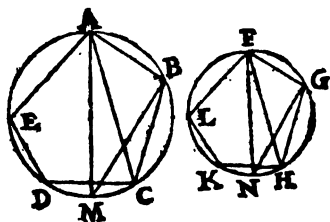
PRO-

PROPOSITION I.

A THEOREM.

Similar Polygons, inscrib'd in Circles are in the same proportion as the Squares of the Diameters of the same Circles.

IF the Polygons
ABCDE, and
FGHKL, inscribed
in Circles, be similar,
they will be in the
same proportion as
the Squares of the
Diameters AM, FN. Draw the Lines BM, GN,
AC, and FH.



Demonstration.

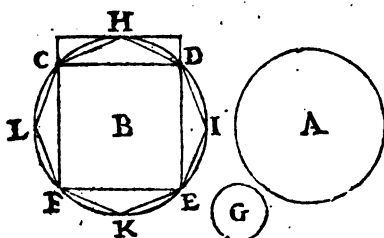
'Tis suppos'd that the Polygons are similar, that is to say, that the Angles B and G are equal, and that AB has the same proportion to BC as FG to GH: from whence I infer, (*by the 6. 6.*) that the Triangles ABC and FGH are equiangular, and that the Angles ACB and FHG are equal; so that likewise (*by the 21. 3.*) the Angles AMB and FNG are equal. But the Angles ABM and FGN, being in a Semicircle, are right Angles, (*by the 31. 3.*) and consequently, the Triangles ABM and FGN are equiangular. Therefore (*by the 4. 6.*) AB has the same proportion to FG, as AM to FN, and (*by the 22.*

6.) if two similar Polygons be describ'd upon AB and FG, as those that are propos'd; and two other similar Polygons upon AM and FN, which shall be two Squares; the Polygon ABCDE will have the same proportion to the Polygon FGH KL; as the Square of AM to the Square of FN.

' This Proposition is necessary to demonstrate
' that which follows.

LEMMA.

If a certain Quantity be less than a Circle, a regular Polygon may be inscrib'd in the same Circle greater than that Quantity.



Suppose the
' Figure A
' to be less than
' the Circle B; a
' regular Poly-
' gon may be
' inscrib'd in
' the same Cir-

' cle, which shall be greater than the Figure
' A. Let the Figure G be the difference be-
' tween the Figure A and the Circle, so that
' the Figures A and G taken together, may be
' equal to the Circle B. Inscribe in the Circle
' B the Square CDEF, (*by the 6. 4.*) and if the
' Square be greater than the Figure A, we shall
' have what we wanted. If it be less, divide
' the four Quarters of the Circle CD, DE, EF
' , and

‘ and FC, each into two equal Parts at the
 ‘ Points H, I, K, L, that so you may have an
 ‘ Octogon. But if the Octogon be still less
 ‘ than the Figure A, subdivide its Arches, and
 ‘ you will have a Polygon of sixteen Sides,
 ‘ afterwards of thirty-two, and then of sixty-
 ‘ four. I say, at length you will have a Poly-
 ‘ gon greater than the Figure A, *i. e.* a Poly-
 ‘ gon whose difference from the Circle is less
 ‘ than that of the Figure A, that is, less than
 ‘ the Figure G.

Demonstration.

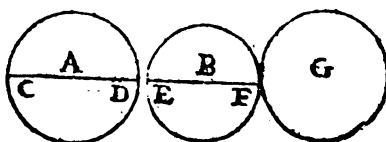
‘ The inscrib’d Square is more than half of
 ‘ the Circle, being half of the Square describ’d
 ‘ about the Circle; and in describing the
 ‘ Octogon you take more than half of the Re-
 ‘ mainder, *i. e.* of the four Segments CHD,
 ‘ DIE, EKF, and CLF. For the Triangle CHD
 ‘ is the half of the Rectangle CO, (*by the 34.*
 ‘ *1.*) therefore it is more than half of the
 ‘ Segment CHD; and the same may be said of
 ‘ all the other Arches. In like manner, in de-
 ‘ scribing the Polygon of sixteen Sides, you
 ‘ take more than half of what was left of the
 ‘ Circle, and so in all the others. Therefore you
 ‘ will leave at last a less Quantity than G. For
 ‘ ’tis evident, that two unequal Quantities being
 ‘ propos’d, if you take away more than half
 ‘ of the greater, and afterwards more than half
 ‘ of what remains, and again more than half
 ‘ of what is still left behind; at length that
 ‘ which remains will be less than the second

Quantity. Suppose the second Quantity to be contain'd in the first an hundred times: 'tis evident, that dividing the first into an hundred Parts, in such sort, that the first Part may have a greater proportion to the second, than two to one; the last will be less than the hundredth Part: so that at last you will obtain a Polygon, which will be less exceeded by the Circle, than the Circle exceeds the Figure A; that is to say, that what will remain of the Circle, when the Polygon is taken away, will be less than G. Therefore the Polygon will be greater than the Figure A.

PROPOSITION II.

A THEOREM.

Circles are in the same proportion as the Squares of their Diameters.



I prove that the Circles A and B are in the same proportion, as the Squares of

CD and EF. Suppose the Figure G to have the same proportion to the Circle B, as the Square of CD to the Square of EF: if the Figure G be less than the Circle A, (*by the preceeding Lemma,*) a regular Polygon may be inscrib'd in the Circle

Circle A, greater than G. Let a similar regular Polygon be also inscrib'd in the Circle B.

Demonstration.

The Polygon of the Circle A will have the same proportion to the Polygon of B, as the Square of CD to the Square of EF, *i. e.* the same as G to the Circle B; but the Quantity G is less than the Polygon inscrib'd in A: accordingly therefore (*by the 14. 5.*) the Circle B must be less than the Polygon inscrib'd in it, which is manifestly false. It must therefore be granted that the Figure G, being less than the Circle A, cannot have the same proportion to the Circle B, as the Square of CD to the Square of EF, and consequently, that the Circle A, cannot have a greater proportion to the Circle B, than the Square of CD to the Square of EF; Nor can it be said to have a less, for then the Circle B would have a greater proportion to the Circle A, and the same Demonstration would be applicable to it.

Coroll. 1. Circles are in the duplicate proportion of that of their Diameters; because the Squares being similar Figures, are in the duplicate proportion of that of their Sides (*by the 20. 6.*)

Coroll. 2. Circles are in the same proportion as the similar Polygons, that are inscrib'd in them.

Coroll. 3. This ought to be well observ'd as a general Rule: When similar Figures, being inscrib'd in others, so that they may approach

still nearer and nearer to them, and at last degenerate into the Figures themselves, are in the same proportion; the Figures that contain them are also in the same proportion. What I would say is this, That similar regular Polygons, inscrib'd in divers Circles, are always in the same proportion as the Squares of the Diameters; and being made of more Sides, so as to approach still nearer and nearer to the Circles, they still retain the same proportion; and the Circles themselves are in the same proportion as the Squares of their Diameters. This manner of measuring round Bodies, by inscribing in them others, is of great use.

The USE.

This being a very general Proposition, enables us to argue about Circles, in the same manner as we do of Squares. For example, we say (*in the 47. 1.*) That in a rectangle Triangle the Square of the Base alone is equal to the Squares of both the Sides taken together. We may say the same of Circles, *i. e.* That the Circle describ'd upon the Base of a Rectangle Triangle, is equal to the Circles, whose Diameters are the Sides. And in the same manner we may augment or diminish Circles, according to what proportion we please. We prove also by it in *Opticks*, that Light decreases in the duplicate proportion of that of the distances of the lucid Bodies,

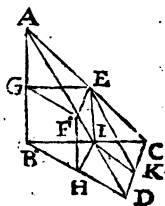
P R O-

PROPOSITION III.

A THEOREM.

Every Pyramid, whose Base is triangular, may be divided into two equal Prisms, which make up more than half of the Pyramid; and into two equal Pyramids.

IN the Pyramid ABCD may be found two equal Prisms, EBFI, and EHKC, which will be greater than half the Pyramid. Divide the six Sides of the Pyramid equally at the Points G, F, E, I, H, K, and draw the Lines EG, GF, FE, EI, HI, FH, IK, and EK.



Demonstration.

In the Triangle ABD, AG has the same proportion to GB as AF to FD, because AB and AD are equally divided in G, and F; therefore (by the 2. 6) GF and BD are Parallels; and GF will be the half of BD, i. e. equal to BH. In like manner, GE and BI, FE and HI, will be parallels, and equal: and (by the 15. 11.) the Planes GFE and BHI will be parallel; and consequently EBFI will be a Prism. The same may be said of the Figure HEKF, which will be also a Prism equal to the other, (by the 40. 11.) the Parallelogram Base HIKD being double the Triangular BHI, (by the 41. 1.)

Secondly,

Secondly, I say, the Pyramids AEFG. and ECKI, are similar and equal.

Demonstration.

The Triangles AFG and FDH are equal, (*by the 8. 1.*) as also FDH and EIK; and likewise AGE, and EIC; and so of all the other Triangles of the Pyramids: therefore the Pyramids are equal, (*by Defin. 10. 11.*) They are also similar to the great Pyramid ABDC: for the Triangles AGE and ECI are similar (*by the 2. 6.*) the Lines GE and BC being Parallels; and the like may be demonstrated of all the other Triangles of the lesser Pyramids.

Lastly, I say the Prisms are more than half of the first Pyramids. For if each was equal to one of the lesser Pyramids, both would be equal to the half of the greater Pyramid. But they are each of them greater than one of those Pyramids; as the Prism GHE contains the Pyramid GBHI, and somewhat more; and that Pyramid is equal and similar to the others, having all their Triangles equal and similar to those of the Pyramid AGFE, as may be easily prov'd by the Parallelism of their Sides: from whence I infer, that the two Prisms taken together, are greater than the two Pyramids; and consequently greater than half of the great Pyramid.

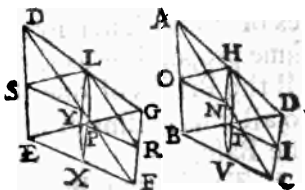
P R O.

PROPOSITION IV.

A THEOREM.

If two Triangular Pyramids of the same height be divided into two Prisms and two Pyramids, and the latter Pyramids subdivided after the same manner; all the Prisms of one Pyramid will have the same proportion to all those of the other, as the Base of one Pyramid to the Base of the other.

IF the two Pyramids ABCD, DEFG, of the same height, and having triangular Bases, be divided into two Prisms and two Pyramids, according to the method laid down in the third Proposition; and the two lesser Pyramids be subdivided after the same manner, and so in order, that having made as many Divisions of one as of the other, you have the same number of Prisms in both; I say, that all the Prisms of one will have the same proportion to all the Prisms of the other, as their Bases.



Demonstration.

The Pyramids being of the same height, the Prisms produc'd by the first Divisions, will have also the same height, because they have each the half of that of their Pyramids. But
Prisms

Prisms of the same height are in the same proportion as their Bases, (*by the Coroll. of the 39. II.*) The Bases BTV and EPX are similar to the Bases BDC and EGF; and having for their Sides the half of those great Bases, they can make but the fourth Part of them; but they are in the same proportion as the great Bases are; therefore the first Prisms will have the same proportion as the great Bases. After the same manner I may prove that the Prisms produc'd by the second Division, *i. e.* of the lesser Pyramids, will be in the same proportion as the Bases of those lesser Pyramids, which are in the same proportion as the great Bases. Therefore all the Prisms of one have the same proportion to all the Prisms of the other, as the Base to the Base.

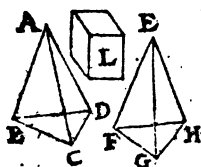
The USE.

* These two Propositions are necessary to
 “ compare Pyramids together, and to take their
 “ Dimensions.

PROPOSITION V.

A THEOREM.

Triangular Pyramids of the same height are in the same proportion as their Bases.



THE Pyramids ABCD and EFGH are in the same proportion as their Bases. For if they were not, one of them, *e. g.* ABCD, would have a greater proportion to the Pyramid

ramid EFGH, than the Base BCD to the Base FGH; so that a Quantity less than ABCD would have the same proportion to the Pyramid EFGH, as the Base BCD to the Base FGH. Divide the Pyramid ABCD after the manner of the third Proposition; divide also the Pyramids that result from the first Division, into two Prisms and two Pyramids, and those again into two other Prisms, continuing the Division as long as there shall be occasion. Since the Prisms of the first Division are more than half of the Pyramid ABCD, (*by the 3.*) and the Prisms of the second Division more than half the remainder, *i. e.* of the two lesser Pyramids, and those of the third Division still more than the half of what is left; it is evident, that so many Divisions may be made, that that which remains shall be less than the excess of the Pyramid ABCD above the Quantity L, that is, that all the Prisms taken together shall be greater than the Quantity L. Make as many Divisions of the Pyramid EFGH, so that you may have as many Prisms as there are in ABCD.

Demonstration.

The Prisms of ABCD have the same proportion to the Prisms of EFGH, as the Base BCD to the Base FGH: but the proportion of the Base BCD to the Base FGH is the same with that of the Quantity L to the Pyramid EFGH: therefore the Prisms of ABCD have the same proportion to the Prisms of EFGH, as the Quantity L to the Pyramid EFGH. But also the

the Prisms of ABCD are greater than the Quantity L : therefore (*by the 14. 5.*) the Prisms contain'd in the Pyramid EFGH would be greater than the same Pyramid EFGH, which is evidently false, because the Part cannot be greater than the Whole. Therefore it must be granted, that no Quantity less than one of the Pyramids can have the same proportion to the other as the Base to the Base ; and consequently neither of the Pyramids can have a greater proportion to the other than the Base has to the Base.

PROPOSITION VI.

A THEOREM.

All sorts of Pyramids of the same height, have the same Proportion as their Bases.



THE Pyramids ABC and DEFG, of the same Height, are in the same proportion as the Bases BC and EFG. Divide the Bases into Triangles.

Demonstration.

The triangular Pyramids AB and DE, being of the same Height, are in the same proportion as their Bases, (*by the 5.*) So also the triangular Pyramids AC and DF are in the same proportion as their Bases. Therefore the Pyramid
ABC

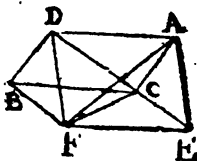
ABC has the same proportion to the Pyramid DEF, as the Base BC to the Base EF, (*by the 12. 5.*) Further, since the Pyramid DEF has the same proportion to the Pyramid ABC, as the Base EF to the Base BC; and again, the Pyramid DG has the same proportion to the Pyramid ABC, as the Base G to the Base BC; the Pyramid DEFG will also have the same proportion to the Pyramid ABC, as the Base EFG to the Base BC.

PROPOSITION. VII.

A THEOREM.

Every Pyramid is the third Part of a Prism being upon the same Base, and of the same height.

Suppose first the triangular Prism AB be propos'd: I say a Pyramid, having one of the Triangles ACE, or BDF for its Base, and being of the same height as the Pyramid ACEF, will be the third Part of the Prism. Draw the three Diagonals AF, DC, FC, of the three Parallelograms.



Demonstration.

The Prism is divided into three equal Pyramids, ACEF, ACFD, and CFBD; therefore each will be the third Part of the Prism. The two first, having for their Bases the Triangles AEF

AEF and **AFD**, which (*by the 34. 1.*) are equal, and for their Height the Perpendicular drawn from the Top **C** to the Plane of their Bases **AF**, will be equal, (*by the preceding.*) The Pyramids **ACFD** and **CFBD**, which for their Bases have the equal Triangles **ADC** and **DCB**, and the same Top **F**, will be also equal, (*by the preceding.*) Therefore one of those Pyramids, *e. g.* **AFCE**, having the same Base **BDF** with the Prism, and the same height, which is the Perpendicular drawn from the Point **F** to the Plane of the Base **ACE**, is the third Part of the same Prism. If the Prism be a Polygon, it must be divided into divers triangular Prisms; and the Pyramid, which has the same Base and the same Height, will be also divided into as many Triangular Pyramids; each of which will be the third Part of its Prism. Therefore (*by the 12. 5.*) the Polygon Pyramid will be the third Part of the Polygon Prism.

PROPOSITION VIII.

A THEOREM.

Similar Pyramids are in the triplicate proportion of that of their homologous Sides.

IF the Pyramids be triangular, compleat the Prisms, which will be also similar, because they will have certain Planes the same with those

those of the Pyramids. But the similar Prisms are in the triplicate proportion of the homologous Sides, (*by Coroll. 4. of the 39. 11.*) therefore the Pyramids, which (*by the preceding,*) are the third Parts of the Prisms, will be in the triplicate proportion of that of their homologous Sides.

If the Pyramids be Polygons, they must be reduc'd to Triangular Pyramids.

PROPOSITION IX.

A THEOREM.

Equal Pyramids have the Heights and Bases reciprocal; and those that have their Heights and Bases reciprocal, are equal.

IF two equal triangular Pyramids be propos'd, make Prisms upon the same Bases, and of the same height. Since every Prism is triple his Pyramid (*by the 7.*) they will also be equal. But equal Prisms have their Bases and Heights reciprocal (*by Coroll. 4. of the 39. 11.*) therefore the Bases and Heights of the Pyramids which are the same with those of the Prisms, will be also reciprocal.

Secondly, if the Bases and Heights of the Pyramids be reciprocal, the Prisms will be equal, as also the Pyramids, which are the third Parts of the Prisms.

If the Pyramids propos'd be Polygons, they must be reduc'd to triangular Pyramids.

Z

Coroll.

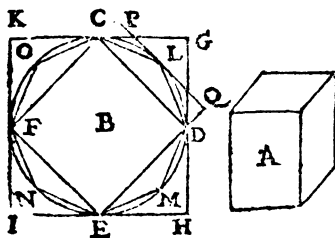
Coroll. Other Propositions may be made concerning Pyramids : as for example ; That Pyramids of the same height, are in the same proportion as their Bases ; and those that have the same Bases, are in the same proportion as their Heights.

The USE

‘ From these Propositions is drawn the manner of Measuring Pyramids, which is, by multiplying their Bases by the third part of their Heights. Other Propositions may also be made, as, That if a Prism be equal to a Pyramid, the Base and the Height of the Prism, with the third part of the Height of the Pyramid, will be reciprocal ; which is as much as to say, that if the Base of the Pyramid has the same proportion to the Base of the Prism, as the Height of the Prism to the third part of the Height of the Pyramid, the Prism and the Pyramid will be equal.

A L E M M A.

If a Quantity less than a Cylinder be propos'd, a Polygon Prism may be inscrib'd in the Cylinder greater than that Quantity.



‘ IF the Quantity
‘ A be less than
‘ the Cylinder, whose
‘ Base is the Circle
‘ B, a Polygon Prism
‘ may be inscrib'd in
‘ the Cylinder great-
‘ er than the Quan-
‘ tity

tity A. The Square CDEF inscrib'd in, and
 GHIK circumscrib'd about, the Circle, CLD
 MENFO is an Octogon inscrib'd. Draw the
 Tangent PLQ, and suppose you had so many
 Prisms as there are Polygon Bases, and all of
 the same height with the Cylinder. That
 which has the circumscrib'd Square for its Base
 will encompass the Cylinder; and that whose
 Base is the inscrib'd Square, will be also in-
 scrib'd in the Cylinder.

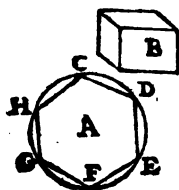
Demonst. Prisms of the same height are in the
 same proportion as their Bases, (*by Coroll. 3.
 of the 39. 11.*) and the inscrib'd Square being
 the half of that which is circumscrib'd, its
 Prism will be the half of the other, and there-
 fore, more than the half of the Cylinder. And
 making the Prism with the Octogon Base, you
 take away more than half of what remain'd of
 the Cylinder, after the Prism of the inscrib'd
 Square was taken from it, because the Trian-
 gle CLD is the Half of the Rectangle CQ.
 And because Prisms of the same height are in
 the same proportion as their Bases, the Prism,
 whose Base is the Triangle CLD, will be the
 half of the Prism, which for its Base has the
 Rectangle DCPQ: it will therefore be more
 than the half of that part of the Cylinder,
 whose Base is the Segment DLC. The same
 may be said of all the other Segments. After
 the same manner I may demonstrate, that
 making a Polygon Prism of sixteen Sides, I
 take away more than half of what remains of

the Cylinder, after the Octogon Prism is taken from it : so that there will remain at last a part of the Cylinder, less than the excess of the Cylinder above the Quantity A. We shall have therefore a Prism inscrib'd in the Cylinder, which shall be less exceeded by the Cylinder than the Quantity A; *i. e.* which shall be greater than the Quantity A. The same way of arguing will hold of the Pyramids inscrib'd in a Cone.

PROPOSITION X.

A THEOREM.

A Cone is the third Part of a Cylinder, having the same Base, and being of the same Height.



IF a Cone and a Cylinder have the Circle A for their Base, and be of the same height, the Cylinder will be triple the Cone. For if the proportion of the Cylinder to the Cone was greater than the triple Proportion, the Quantity B less than the Cylinder would have the same proportion to the Cone as three to one : and (*by the preceding Lemma*) a Polygon Prism may be inscrib'd in the Cylinder greater than the Quantity B. Suppose that which has for its Base the Polygon CDEFGH to be such an

an one; Make also upon the same Base a Pyramid inscrib'd in the Cone.

Demonst. The Cylinder, the Cone, the Prism, and the Pyramid, are of the same Height; therefore the Prism is the triple of the Pyramid, (*by the 7.*) But the Quantity B is also the triple of the Cone; therefore the Prism has the same proportion to the Pyramid, as the Quantity B to the Cone: and (*by the 14. 5.*) the Prism being greater than the Quantity B, the Pyramid would be greater than the Cone, in which it is inscrib'd, which is impossible.

But if it be said, that the Cone has a greater proportion to the Cylinder than one to three, the same Method may be made use of to demonstrate the contrary.

PROPOSITION XI.

A THEOREM.

Cylinders and Cones of the same height, are in the same proportion as their Bases.

LET two Cones, or two Cylinders, of the same height, be propos'd, having for their Bases



the Circles A and B: I say, they

they are in the same proportion as their Bases. For if not, one of them, *e. g.* the Cylinder A, would have a greater proportion to the Cylinder B, than the Base A has to the Base B; suppose then that the Quantity L, less than the Cylinder A, has the same proportion to the Cylinder B as the Base A, to the Base B. Therefore a Polygon Prism may be inscrib'd in the Cylinder A, which shall be greater than the Quantity L. Suppose it *that* therefore, whose Base is the Polygon CDEF; and inscribe a similar Polygon GHIK in the Base B, which is also the Base of a Cylinder of the same height.

Demonstr. The Prisms of A and B are in the same proportion as their Polygon Bases, (*by Coroll. 4. of the 39. 11.*) and the Polygons are in the same proportion as the Circles, (*by Coroll. 2. of the 12.*) therefore the Prism A has the same proportion to the Prism B, as the Circle A to the Circle B. But as the Circle A to the Circle B, so is the Quantity L to the Cylinder B; therefore as the Prism A to the Prism B, so is the Quantity L to the Cylinder B. But the Prism A is greater than the Quantity L, and consequently (*by the 14. 5.*) the Prism B, inscrib'd in the Cylinder B, would be greater than its Cylinder, which is impossible. Therefore neither of the Cylinders has a greater Proportion to the other, than its Base to the other's Base.

Coroll.

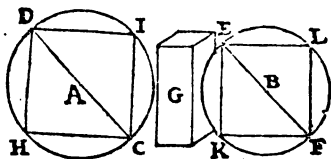
Coroll. Cylinders are triple the Cones, of the same height, therefore Cones of the same height are in the same proportion as their Bases.

PROPOSITION. XII.

A THEOREM.

Cylinders and Cones, that are similar, are in the triplicate proportion of that of the Diameters of their Bases.

LET two Cones
or two Cylinders, that are similar, be propos'd, having the Circles, A and B for their Bases;



I say, that the proportion of the Cylinder A to the Cylinder B is the triplicate proportion of that of the Diameter DC to the Diameter EF. For, if it be not the triplicate proportion; let the Quantity G, less than the Cylinder A, be to the Cylinder, in the triplicate proportion of that of the Diameter DC to the Diameter EF; and inscribe a Prism in the Cylinder A greater than G, and another similar to it in the Cylinder B: they will be of the same height with the Cylinder, because similar Cylinders have their Heights and the Diameters of their Bases proportional, as well as Prisms, (*by Defn. 22. 11.*)

Z 4

Demon-

Demonstr. The Diameter DC has the same proportion to the Diameter EF as the Side DI to the Side EL, or as DC to EF, as I have shown in the first. But similar Prisms are in the triplicate proportion of that of their homologous Sides, (by the Coroll. 4. of 39. 11.) therefore the Prism A is to the Prism B in the triplicate proportion of that of DC to EF. But we suppos'd that the Quantity G in respect of the Cylinder B was in the triplicate proportion of that of DC to EF; therefore the Prism A will have the same proportion to the Prism B, as the Quantity G to the Cylinder B; and (by the 14. 5.) the Prism A being greater than the Quantity G, the Prism B, inscrib'd in the Cylinder B, will be greater than the Cylinder B, which is impossible. Therefore similar Cylinders are in the triplicate proportion of that of the Diameters of their Bases.

Cones are the third Parts of Cylinders (by the 10.) therefore similar Cones are in the triplicate proportion of that of the Diameters of their Bases.

PROPOSITION XIII.

A THEOREM.

If a Cylinder be cut by a Plane, that is parallel to its Base, the Parts of its Axis will be in the same proportion as the Parts of the Cylinder.



LET the Cylinder AB be cut by the Plane DC parallel to its Base: I say, the Cylinder AF will have the same proportion to the Cylinder FB, as the Line AF to the Line FB.

FB. Draw the Line BG perpendicular to the Plane of the Base A. Draw also upon the Planes of the Circles DC and A the Lines FE and AG.

Demonstration.

The Plane of the Triangle BAG cuts the parallel Planes A and DC; therefore the Sections FE and AG are parallel, (*by the 16. 11.*) So that AF has the same proportion to FB, as the Height GE to EB. Take any aliquot Part of EB; and having divided GE and EB into Parts equal to it, draw so many Planes parallel to the Base A; then will you have so many Cylinders of the same height; which having their Bases and Heights equal, will be equal, (*by the 11.*)

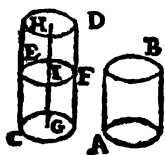
Further, the Lines AF and FB will be divided after the same manner as EG and EB, (*by the 17. 11.*) so that the Line AF will as oft contain any aliquot Part of the Line FB, as the Cylinder AF contains the like aliquot Part of the Cylinder FB; therefore the Parts of the Cylinder will be in the same proportion as the Parts of their Axis.

Coroll. The Parts of the Perpendicular are in the same proportion as the Parts of the Cylinder.

PROPOSITION XIV.

A THEOREM.

Cylinders and Cones, having the same Bases, are in the same proportion as their Heights.



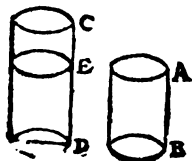
TWO Cylinders of equal Bases being propos'd, as AB and CD, cut in the greater a Cylinder of the same Height with the less, drawing a Plane EF parallel to its Base. 'Tis evident (*by the II.*) that the Cylinders CF and AB are equal; and that CF has the same proportion to CD, as GI to GH, or (*by the Coroll. of the preceeding*) as the Height of CF to the Height of CD; therefore AB has the same proportion to CD, as the Height of CF or AB to the Height of GD.

Cones being the third Part of Cylinders, if their Bases be equal, will be also in the same proportion as their Heights.

PROPOSITION XV.

A THEOREM.

Cylinders and Cones, that are equal, have their Bases and Heights reciprocal: and those that have their Bases and Heights reciprocal, are equal.



IF the Cylinders AB and CD be equal, the Base B will have the same proportion to the Base D, as the Height CD to the Height

Height AB. Take the Height DE equal to the Height AB.

Demonstr. The Cylinder AB has the same proportion to the Cylinder DE, of the same height, as the Base B to the Base D, (*by the 11.*) But as the Cylinder AB is to the Cylinder DE, so is the Cylinder CD (equal to AB) to the Cylinder DE; *i. e.* so is the Height CD to the Height AB or DE. Therefore as the Base B to the Base D, so is the Height CD to the Height AB.

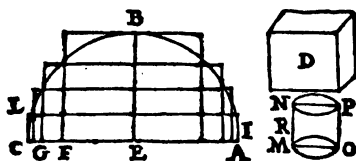
Secondly, If the Base B has the same proportion to the Base D, as the Height CD to the Height AB, the Cylinders AB and CD will be equal. For the Cylinder AB is in the same proportion to the Cylinder DE, as the Base B to the Base D: and the Cylinder CD will have the same proportion to DE, as the Height CD to the Height DE: therefore AB has the same proportion to DE, as CD to DE; and (*by the 9. 5.*) the Cylinders AB and CD will be equal.

“ The 16. and 17. Propositions are very
“ difficult, and of no other use but to prove the
“ 18. which may more easily be done by the
“ following Lemma's.

PRO-

L E M M A I.

*If a Quantity be propos'd less than a Sphere, Cy-
linders of the same Height may be inscrib'd in
the same Sphere greater than that Quantity.*



Suppose ABC
be a great
Semicircle of the
Sphere, whereof
we treat, and the
Quantity D to be
the Quantity less than that Sphere: I say,
several Cylinders of the same height may be
inscrib'd in the Sphere, which taken together
will be greater than the Quantity D. For if
the Hemisphere exceed the Quantity D, it
will exceed it by some Magnitude; let it then
be the Cylinder MP, so that the Quantities
D and MP taken together may be equal to the
Hemisphere. Make a greater Circle of the
Sphere to have the same proportion to the
Base MO, as the Height MN to the Height
R. Then divide the Line EB into as many e-
qual Parts as you please, each being less than
R: and drawing Parallels to the Line AG,
describe the inscrib'd and circumscrib'd
Parallelograms. The number of the cir-
cumscrib'd will exceed that of the inscrib'd
by one. But all the Rectangles circum-
scrib'd will surpass all the inscrib'd by the
little

' little Rectangles through which the Cir-
 ' cumference of the Circle passes; all which
 ' taken together are equal to the Rectangle
 ' AL. I imagine then the Semicircle to be turn'd
 ' about upon the Diameter EB; the Semicircle
 ' will by that Motion describe a Hemisphere,
 ' and the Rectangles inscrib'd so many Cylinders
 ' inscrib'd in the Hemisphere; and the circum-
 ' scrib'd, other Cylinders circumscrib'd.

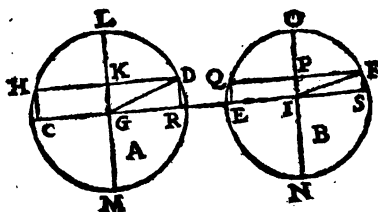
Demonstr. ' The circumscrib'd Cylinders sur-
 ' pass the inscrib'd more than the Hemisphere
 ' surpasses the same inscrib'd Cylinders, it be-
 ' ing contain'd within the circumscrib'd Cy-
 ' linders. But the circumscrib'd surpasses the
 ' inscrib'd by the Cylinder AL: therefore
 ' the Hemisphere will surpass those inscrib'd
 ' Cylinders by less than the Cylinder de-
 ' scrib'd by the Rectangle AL. But the Cy-
 ' linder AL is less than the Cylinder MP: for
 ' there is the same proportion of a great Circle
 ' of the Sphere, which is the Base of the Cy-
 ' linder AL, to MQ, as of MN to R; there-
 ' fore (by the preceeding) a Cylinder, which
 ' should have a great Circle of the Sphere for
 ' its Base, and the Height R, would be equal
 ' to the Cylinder MP: but the Cylinder AL,
 ' tho' it have the same Base, yet its Height CL
 ' is less than R; therefore the Cylinder AL is
 ' less than the Cylinder MP. Consequently
 ' the Hemisphere that exceeds the Quantity D
 ' by the Cylinder MP, and the inscrib'd Cy-
 ' linders by a Quantity less than AL; exceeds
 the

‘ the inscrib’d Cylinders by less than it exceeds
 ‘ the Quantity D; therefore the Quantity D
 ‘ is less than the Cylinders inscrib’d in the
 ‘ Hemisphere.

‘ That which I have said of the Hemisphere,
 ‘ is applicable to an entire Sphere.

LEMMA II.

Similar Cylinders, inscrib’d in two Spheres, are in the triplicate proportion of the Diameters of the Spheres.



‘ IF two similar
 ‘ Cylinders CD
 ‘ and EF be in-
 ‘ scrib’d in the
 ‘ Spheres A and
 ‘ B, they will
 ‘ be in the tri-
 ‘ plicate proportion of the Diameters LM and
 ‘ NO. Draw the Lines GD and IF.

Demonstration.

‘ The right Cylinders CD and EF are simi-
 ‘ lar; therefore HD has the same proportion to
 ‘ DR as QF to FS, as also KD has the same
 ‘ proportion to DG as PF to FI. Consequently
 ‘ the Triangles GDK and IFP are similar, (by
 ‘ the 6. 6.) therefore KD has the same propor-
 ‘ tion to PF, as GD to IF, as LM to ON. But
 ‘ the similar Cylinders CD and EF are in the
 ‘ triplicate proportion of KD and PF, the
 Semidi-

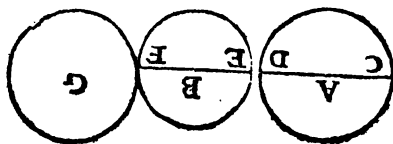
- Semidiameters of their Bases (by the 12.)
- therefore the similar Cylinders CD and EF,
- inscrib'd in the Spheres A and B, are in the
- triplicate proportion of the Diameters of the
- Spheres.

PROPOSITION XVIII.

A THEOREM.

Spheres are in the triplicate proportion of their Diameters.

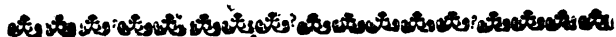
TH E Spheres A and B are in the triplicate proportion of that of their Diameters CD and EF. For if not, one of the Spheres, suppose A, will be in a greater proportion to B, than the Triplicate of that of the Diameters CD and EF; therefore the Quantity G less than the Sphere A, will be to the Sphere B, in the triplicate proportion of that of CD to EF; and then some Cylinders may (*according to Lem. 1.*) be inscrib'd in the Sphere A, greater than the Quantity G. Inscribe an equal number of Cylinders in the Sphere B, similar to those in the Sphere A.



Demon-

Demonstr. The Cylinders of the Sphere A to those of the Sphere B are in the triplicate proportion of that of CD to EF, but the Quantity G to the Sphere B, is also in the triplicate proportion of that of CD to EF: therefore the Cylinders of the Sphere A have the same proportion to the similar Cylinders of the Sphere B, as the Quantity G to the Sphere B. Consequently the Cylinders of A being greater than the Quantity G, the Cylinders of B, *i. e.* inscrib'd in the Sphere B, will be greater than the Sphere B, which is impossible. Therefore the Spheres A and B are in the triplicate proportion of that of their *Diameters*.

Coroll. Spheres are in the same proportion as the Cubes of their Diameters; because Cubes being similar Solids, are in the triplicate proportion of their Sides, (*by the 33. 11.*)



The E N D.

